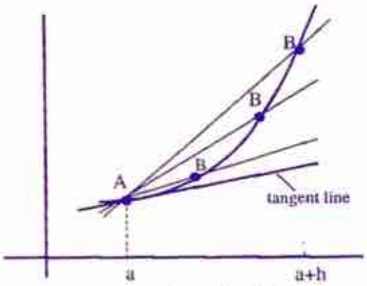
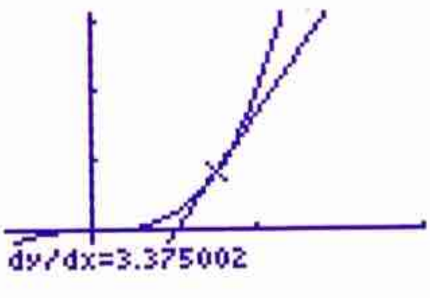
ADVANCED PLACEMENT MATHEMATICS

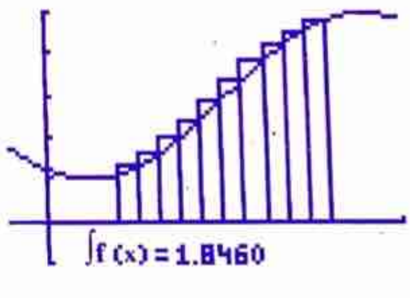
# PREPARING FOR THE (AB) AP CALCULUS EXAMINATION




$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$


$\int_a^b f(x)\,dx = \lim_{n \to \infty} [f(x_1)\Delta x_1 + f(x_2)\Delta x_2 + f(x_3)\Delta x_3 + \ldots + f(x_n)\Delta x_n]$





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# PREPARING FOR THE (AB) AP CALCULUS EXAMINATION

6 exams (2 parts each)

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# Preface

This workbook is intended for students preparing to take the Advanced Placement Calculus AB Examination. It contains six practice tests that are based on the course description published by the College Board. We have tried to make each of the six tests in this workbook as much like the actual AP Exam as possible. For example, in the appropriate sections, there are questions that require students to make decisions about whether to use the graphing calculator a lot, a little, or not at all. In order to provide a greater supply of this type problem, our exams require the use of a calculator in about half the problems of Section I Part B, and all of Section II Part A.

Each student is expected to have a graphing calculator that has the capability to:
(1) plot the graph of a function within an arbitrary viewing window,
(2) find the zeros of a function (solve equations numerically),
(3) calculate the derivative of a function numerically, and
(4) numerically calculate the value of a definite integral.

In the free-response sections, solutions obtained using one of these four capabilities need only show the setup. Solutions using other calculator capabilities must show the mathematical steps that lead to the answer. In either case, a correct answer alone will not receive full credit.

As in the *AP Course Description for Mathematics*, our examinations are in two sections of equal weight. Section I is all multiple-choice and Section II is all free-response.

1. Section I Part A (28 questions in 55 minutes). Calculators may not be used in this part of the exam.
2. Section I Part B (17 questions in 50 minutes). Calculators are allowed.
3. Section II Part A (3 questions in 45 minutes). Calculators are required
4. Section II Part B (3 questions in 45 minutes). Calculators may not be used. Students may go back and work on Part A without a calculator if time permits.

We have tried to create the problems in the spirit of *calculus reform*. Calculus reform implies a change in the mode of instruction as well as increased focus on concepts and less attention to symbolic manipulation; emphasis on modeling and applications; use of technology to explore and deepen understanding of concepts; projects and cooperative learning. We have included questions where functions are defined graphically and numerically, as well as symbolically, in order to give the students more practice in this type of analysis.

We wish to thank the members of the Phillips Academy Mathematics Department for their generous contributions of ideas, problems and advice. Their valuable assistance in testing the problems in the classroom has made us quite confident about the validity of the exams. Robert Clements of Phillips Exeter Academy provided excellent editorial assistance and insightful comments.

Special thanks go to our wives, Ann and Helen, for their patience, understanding and encouragement.

In the hope of providing future students with a better workbook, the authors welcome your suggestions, corrections, problems of all sorts, and feedback in general. Please send your comments to:

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# For The Student

There are six examinations in this workbook. Use them as suggested by your teacher, but about two weeks prior to the AP Exam you should try to find a three hour and thirty minute block of time to work through one entire exam. Each part of the exam should be carefully timed. Allow fifty-five minutes for Section I Part A, fifty minutes for Section I Part B, and ninety minutes for Section II. Take a ten minute break between Part A and Part B and also between Part B and Section II. This will give you a good measure of the topics that need more intensive review as well as give you a feel for the energy and enthusiasm needed on a three hour and fifteen minute exam. Repeat the above routine on a second exam four or five days before the AP to check your progress.

The questions on these exams are designed to be as much like the actual AP Exams as possible. However, we have included a greater percentage of medium level and difficult problems and fewer easy ones, in order to help you gain stamina and endurance. If you do a satisfactory job on these exams, then you should be confident of doing well on the actual AP Exam.

The answers to the multiple-choice questions and selected free-response questions are in the back of the workbook. A complete solution manual for all the problems is available from Venture Publishing. No matter how much of an exam you do at one sitting, we strongly urge you to check your answers when you are finished, not as you go along. You will build your confidence if you DO NOT use the "do a problem, check the answer, do a problem" routine.

The following is a list of common student errors:

1. If $f'(c)=0$, then $f$ has a local maximum or minimum at $x=c$.
2. If $f''(c)=0$, then the graph of $f$ has an inflection point at $x=c$.
3. If $f'(x)=g'(x)$, then $f(x)=g(x)$.
4. $\frac{d}{dx}f(y)=f'(y)$
5. Volume by washers is $\int_a^b (R-r)^2 dx$.
6. Not expressing answers in correct units when units are given.
7. Not providing adequate justification when justification is requested.
8. Wasting time erasing bad solutions. Simply cross out a bad solution after writing the correct solution.
9. Listing calculator results without the supporting mathematics. Recall that a calculator is to be used primarily to:
   a) graph functions,
   b) compute numerical approximations of a derivative and definite integral,
   c) solve equations.
10. Not answering the question that has been asked. For example, if asked to find the maximum value of a function, do not stop after finding the $x$-value where the maximum value occurs.

## Table of Contents

EXAM I

Section I Part A . . . . . . . . . . . . . . . . . 1
Section I Part B . . . . . . . . . . . . . . . . . 11
Section II Part A . . . . . . . . . . . . . . . . 18
Section II Part A . . . . . . . . . . . . . . . . 22

EXAM II

Section I Part A . . . . . . . . . . . . . . . . . 25
Section I Part B . . . . . . . . . . . . . . . . . 35
Section II Part A . . . . . . . . . . . . . . . . 42
Section II Part A . . . . . . . . . . . . . . . . 46

EXAM III

Section I Part A . . . . . . . . . . . . . . . . . 49
Section I Part B . . . . . . . . . . . . . . . . . 59
Section II Part A . . . . . . . . . . . . . . . . 66
Section II Part A . . . . . . . . . . . . . . . . 70

EXAM IV

Section I Part A . . . . . . . . . . . . . . . . . 73
Section I Part B . . . . . . . . . . . . . . . . . 83
Section II Part A . . . . . . . . . . . . . . . . 90
Section II Part A . . . . . . . . . . . . . . . . 94

EXAM V

Section I Part A . . . . . . . . . . . . . . . . . 97
Section I Part B . . . . . . . . . . . . . . . . . 107
Section II Part A . . . . . . . . . . . . . . . . 114
Section II Part A . . . . . . . . . . . . . . . . 118

EXAM VI

Section I Part A . . . . . . . . . . . . . . . . . 121
Section I Part B . . . . . . . . . . . . . . . . . 131
Section II Part A . . . . . . . . . . . . . . . . 139
Section II Part A . . . . . . . . . . . . . . . . 143

ANSWERS . . . . . . . . . . . . . . . . . . . . 146

INDEX OF TOPICS . . . . . . . . . . . . . . . . 149

EXAM I
CALCULUS AB
SECTION I PART A
MULTIPLE CHOICE
NO CALCULATORS
Time–55 minutes
Number of questions–28

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number. $x-2>0$
$x>2$

1. If $f'(x) = \ln(x-2)$, then the graph of $y = f(x)$ is decreasing if and only if

(A) $2 < x < 3$ (B) $0 < x$ (C) $0 < x < 1$ (D) $x > 1$ (E) $x > 2$

$f'(x)=0$
$x-2=1$
$x=3$

$-$ $+$
$2$ $3$

Ans

2. For $x \neq 0$, the slope of the tangent to $y = x\cos x$ equals zero whenever

(A) $\tan x = -x$

(B) $\tan x = \frac{1}{x}$

(C) $\tan x = x$

(D) $\sin x = x$

(E) $\cos x = x$

$y' = \cos x + x(-\sin x) = 0$
$\cos x - x\sin x = 0$
$x\sin x = \cos x$
$x = \cot x$
$x = \frac{1}{\tan x}$
$\tan x = \frac{1}{x}$

Ans

3. The function $F$ is defined by

$F(x) = G[x + G(x)]$

where the graph of the function $G$ is shown at the right.

The approximate value of $F'(1)$ is

(A) $\frac{7}{3}$

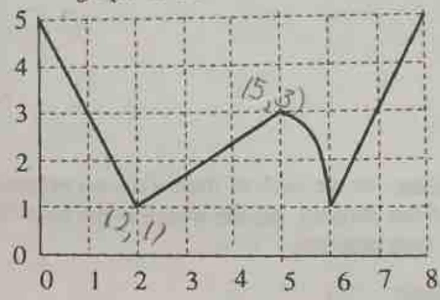
(B) $\frac{2}{3}$

(C) $-2$

(D) $-1$

(E) $-\frac{2}{3}$

graph of the function $G$




$(0, 5)$ $\frac{1-5}{2-0} = \frac{-4}{2} = -2$

$F'(x) = G'[x+G(x)](1+G'x)$

$F'(1) = G'[1+G(1)](1+G'(1))$

$= G'[4](1-2)$

$= G'(4)(-1)$

$= \frac{2}{3}(-1) = -\frac{2}{3}$

$\frac{5-1}{5-2} = \frac{2}{3}$

Ans

4. $\int_{2}^{6} \left(\frac{1}{x} + 2x\right) dx =$

(A) $\ln 4 + 32$

(B) $\ln 3 + 40$

(C) $\ln 3 + 32$

(D) $\ln 4 + 40$

(E) $\ln 12 + 32$

$\int_2^6 (x^{-1} + 2x)\,dx$

$= \left[\ln x + x^2\right]_2^6$

$= \ln 6 + 36 - \ln 2 - 4$

$= \ln 3 + 32$

Ans

5. A relative maximum of the function $f(x) = \frac{(\ln x)^2}{x}$ occurs at

(A) 0

(B) 1

(C) 2

(D) $e$

(E) $e^2$

$x \neq 0$

$x > 0$

$f'(x) = \frac{x[2(\ln x)(1/x)] - (\ln x)^2(1)}{x^2}$

$\frac{2\ln x - (\ln x)^2}{x^2} = 0$

$2\ln x - (\ln x)^2 = 0$

$\ln x(2 - \ln x)$

$\ln x = 0$ $\quad 2 - \ln x = 0$

$x = 1$ $\quad \ln x = 2$

$x = e^2$

$-$ $+$ $-$

0 1 $e^2$

Ans

6. Use a right-hand Riemann sum with 4 equal subdivisions to approximate the integral $\int_{-1}^{3} |2x-3|\, dx$. – to graph ;

$\Delta x =$

$\frac{3+1}{4}\left[ f \quad + f(0) + f(1) + f(2) + f(3) \right]$

$5+1+1+3$

$= 8$

(A) 13

(B) 10

(C) 8.5

(D) 8

(E) 6

Ans

---

7. An equation of the line tangent to the graph of $y = x^3 + 3x^2 + 2$ at its point of inflection is

$y' = 3x^2 + 6x \qquad y' = -3$

$y'' = 6x + 6 = 0$

$6x = -6$

$x = -1 \qquad y = 4$

$y - 4 = -3(x+1)$

$y = -3x + 1$

(A) $y = -3x + 1$

(B) $y = -3x - 7$

(C) $y = x + 5$

(D) $y = 3x + 1$

(E) $y = 3x + 7$

Ans

---

8. $\int \cos(3-2x)\, dx =$

$\frac{\sin(3-2x)}{-2}$

(A) $\sin(3-2x) + C$

(B) $-\sin(3-2x) + C$

(C) $\frac{1}{2}\sin(3-2x) + C$

(D) $-\frac{1}{2}\sin(3-2x) + C$

(E) $-\frac{1}{5}\sin(3-2x) + C$

Ans

9. What is $\lim_{x\to\infty} \dfrac{\sqrt{9x^2+2}}{4x+3}$ ?

(A) $\frac{3}{2}$ (B) $\frac{3}{4}$ (C) $\frac{\sqrt{2}}{3}$ (D) 1 (E) The limit does not exist.

$$\frac{\sqrt{\frac{9x^2+2}{x^2}}}{4+\frac{3}{x}} = \frac{\sqrt{9+\frac{2}{x^2}}}{4+\frac{3}{x}} = \frac{\sqrt{9+0}}{4+0} = \frac{3}{4}$$

Ans

10. Let the first quadrant region enclosed by the graph of $y = \dfrac{1}{x}$ and the lines $x = 1$ and $x = 4$ be the base of a solid. If cross sections perpendicular to the $x$-axis are semicircles, the volume of the solid is

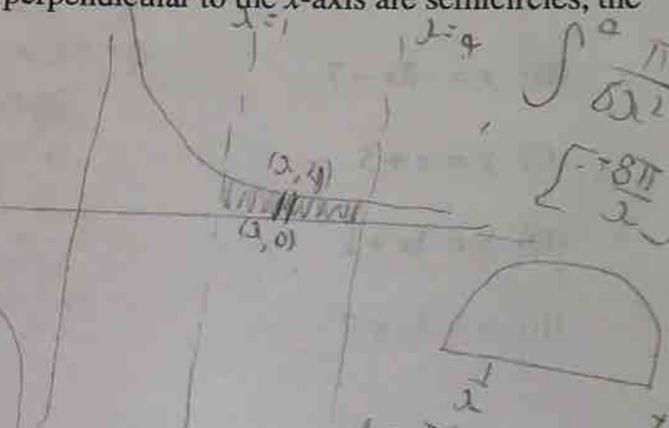
(A) $\frac{3\pi}{64}$ units$^3$

(B) $\frac{3\pi}{32}$ units$^3$

(C) $\frac{3\pi}{16}$ units$^3$

(D) $\frac{3\pi}{8}$ units$^3$

(E) $\frac{3\pi}{4}$ units$^3$




$\frac{\pi}{8}x^{-2}$

Volume $= \int_1^4 \frac{\pi\left(\frac{1}{2x}\right)^2}{2} = \int_1^4 \left(\frac{\pi}{8x^2}\right) = \left[\frac{-\pi}{8x}\right]_1^4 = \frac{-\pi}{32} + \frac{\pi}{8} = \frac{-\pi+4\pi}{32} = \frac{3\pi}{32}$

$A = \frac{\pi r^2}{2}$

$\int^4 \frac{\pi}{8x^2}$ $\left[\frac{-8\pi}{x}\right]^4$

Ans

11. Let $f(x) = \ln x + e^{-x}$. Which of the following is TRUE at $x = 1$?

(A) $f$ is increasing

(B) $f$ is decreasing

(C) $f$ is discontinuous

(D) $f$ has a relative minimum

(E) $f$ has a relative maximum

$f'(x) = \frac{1}{x} - e^{-x} \neq 0$

$f'(1) = 1 - \frac{1}{e}$

$\frac{1}{x} = \frac{1}{e^x}$

$x = e^x$

Ans

12. Suppose $F(x) = \int_0^{x^2} \frac{1}{2+t^3}\,dt$ for all real $x$, then $F'(-1) =$

(A) 2 (B) 1 C) $\frac{1}{3}$ (D) $-2$ (E) $-\frac{2}{3}$

$F'(x) = \frac{1}{2+x^6}(2x)$

$F'(-1) = \frac{-2}{2+1} = -\frac{2}{3}$

Ans

13. What is the average (mean) value of $2t^3 - 3t^2 + 4$ over the interval $-1 \le t \le 1$ ?

(A) 0

(B) $\frac{7}{4}$

(C) 3

(D) 4

(E) 6

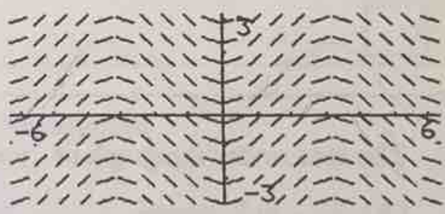
$2-3+4$

$-2-3+4=-1$

$\frac{1}{2}\int_{-1}^{1}(2t^3-3t^2+4)\,dx$

Ans

14. The slope field for a differential equation $\frac{dy}{dx} = f(x,y)$ is given in the figure. The slope field corresponds to which of the following differential equations?




(A) $\frac{dy}{dx} = \tan x \cdot \sec x$

(B) $\frac{dy}{dx} = \sin x$

(C) $\frac{dy}{dx} = \cos x$

(D) $\frac{dy}{dx} = -\sin x$

(E) $\frac{dy}{dx} = -\cos x$

Ans

15. What is $\lim_{x \to 1} \dfrac{\sqrt{x}-1}{x-1}$ ? $\cdot \dfrac{\sqrt{x}+1}{\sqrt{x}+1} = \dfrac{x-1}{(x-1)(\sqrt{x}+1)} = \dfrac{1}{\sqrt{x}+1} = \dfrac{1}{2}$

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) $\frac{3}{2}$

(E) The limit does not exist.

Ans $\boxed{\phantom{x}}$

16. If $y = \cos^2 x - \sin^2 x$, then $y' =$

(A) $-1$ $2\cos x(-\sin x) - 2\sin x(\cos x)$

(B) 0 $-2\sin x\cos x - 2\sin x\cos x$

(C) $-2(\cos x + \sin x)$ $-4\sin x\cos x$

(D) $2(\cos x + \sin x)$

(E) $-4(\cos x)(\sin x)$

Ans $\boxed{\phantom{x}}$

17. The area under the graph of $y = 4x^3 + 6x - \dfrac{1}{x}$ on the interval $1 \le x \le 2$ is

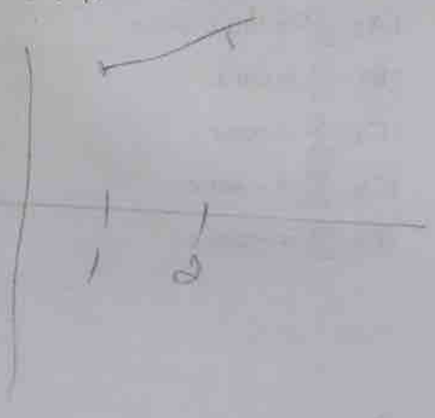
(A) $32 - \ln 2$ units$^2$ $4+6-1$

(B) $30 - \ln 2$ units$^2$ $32+12-\frac{1}{2}$

(C) $24 - \ln 2$ units$^2$

(D) $\dfrac{99}{4}$ units$^2$

(E) 21 units$^2$




$\int_1^2 4x^3+6x-\frac{1}{x}$

$= \left[x^4+3x^2-\ln x\right]_1^2$

$= 16+12-\ln 2$
$-1-3+\ln 1$ $= 24+\ln 1-\ln 2$

Ans $\boxed{\phantom{x}}$

18. $\int \frac{x-2}{x-1}\,dx =$

$\int \frac{x-2}{x-1}$

(A) $-\ln|x-1| + C$

(B) $x + \ln|x-1| + C$

(C) $x - \ln|x-1| + C$

(D) $x + \sqrt{x-1} + C$

(E) $x - \sqrt{x-1} + C$

$\frac{x}{x-1} - \frac{2}{x-1}$

$= 1 + x - \frac{1}{x-1}$

$= x - \ln|x-1|$

$1 + \frac{1}{x-1} - \frac{2}{x-1} = 1 - \frac{1}{x-1}$

Ans

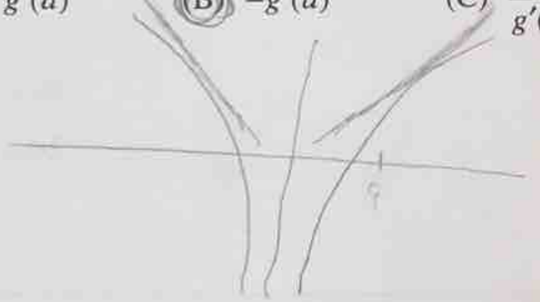
---

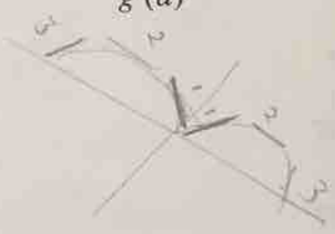
even

19. Suppose that $g$ is a function with the following two properties: $g(-x) = g(x)$ for all $x$, and $g'(a)$ exists. Which of the following must necessarily be equal to $g'(-a)$?

(A) $g'(a)$ (B) $-g'(a)$ (C) $\frac{1}{g'(a)}$ (D) $-\frac{1}{g'(a)}$ (E) none

continuous @ a



Ans

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20. An equation for a tangent to the graph of $y = \text{Arctan}\,\frac{x}{3}$ at the origin is:

(A) $x - 3y = 0$

(B) $x - y = 0$

(C) $x = 0$

(D) $y = 0$

(E) $3x - y = 0$

$y = \tan^{-1}\frac{x}{3}$ $(0,0)$

$y' = \frac{\frac{1}{3}}{1+(\frac{x}{3})^2} = \frac{1}{3+\frac{x^2}{3}}$

$y'(0) = \frac{1}{3}$

$y - 0 = \frac{1}{3}(x - 0)$

$y = \frac{1}{3}x$ $3y = x$

$-3y + x = 0$

Ans

21. If $f(x) = \begin{cases} x^2 + 4 & \text{for } 0 \le x \le 1 \\ 6 - x & \text{elsewhere} \end{cases}$ then $\int_0^3 f(x)\,dx$ is a number between

(A) 0 and 5

(B) 5 and 10

(C) 10 and 15

(D) 15 and 20

(E) 20 and 25

$\int_0^1 x^2+4 + \int_1^3 6-x$

Ans

22. $\frac{d}{dx}\left(\ln e^{3x}\right) =$

(A) 1

(B) 3

(C) $3x$

(D) $\frac{1}{e^{3x}}$

(E) $\frac{3}{e^{3x}}$

$y = \ln e^{3x}$

$y' = \frac{3e^{3x}}{e^{3x}} = 3$

Ans

23. If $g'(x) = 2g(x)$ and $g(-1) = 1$, then $g(x) =$

(A) $e^{2x}$

(B) $e^{-x}$

(C) $e^{x+1}$

(D) $e^{2x+2}$

(E) $e^{2x-2}$

$\frac{dy}{dx} = 2y$

$\int \frac{dy}{y} = \int 2\,dx$

$\ln y = 2x + c$

$y = Ce^{2x}$

$(-1, 1)$

$1 = Ce^{-2}$

$C = \frac{1}{e^{-2}}$

$C = e^2$

$y = e^{2x}e^2 = e^{2x+2}$

Ans

24. The acceleration at time $t > 0$ of a particle moving along the $x$-axis is $a(t) = 3t + 2$ ft/sec$^2$. If at $t = 1$ seconds the velocity is 4 ft/sec and the position is $x = 6$ feet, then at $t = 2$ seconds the position $x(t)$ is

(A) 8 ft (B) 11 ft (C) 12 ft (D) 13 ft (E) 15 ft

Ans

25. The approximate value of $y = \sqrt{3 + e^x}$ at $x = 0.08$, obtained from the tangent to the graph at $x = 0$, is

(A) 2.01

(B) 2.02

(C) 2.03

(D) 2.04

(E) 2.05

$y' = \frac{1}{2}(3+e^x)^{-1/2}(e^x)$

$(0, 2)$

$y - 2 = \frac{1}{4}(x - 0)$

$y - 2 = \frac{1}{4}(0.08)$

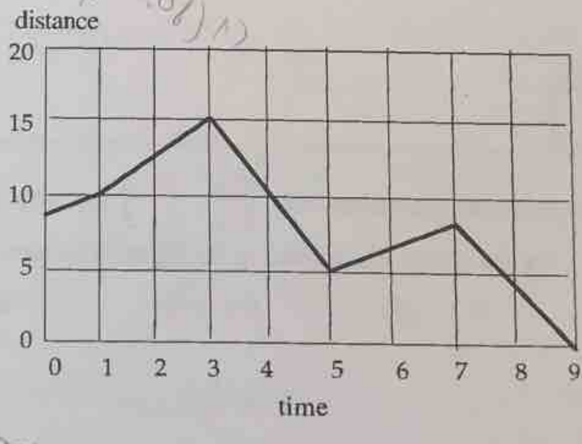
$y - 2 = 0.02$

$y = 2.02$

Ans

26. A leaf falls from a tree into a swirling wind. The graph at the right shows the vertical distance (feet) above the ground plotted against time (seconds).

According to the graph, in what time interval is the speed of the leaf the greatest?




(A) $1 < t < 3$

(B) $3 < t < 5$

(C) $5 < t < 7$

(D) $7 < t < 9$

(E) none of these

Speed = magnitude of velocity

Slope of graph greatest

Ans

27. Water is flowing into a spherical tank with 6 foot radius at the constant rate of $30\pi$ cu ft per hour. When the water is $h$ feet deep, the volume of water in the tank is given by

$$V = \frac{\pi h^2}{3}(18 - h).$$

What is the rate at which the depth of the water in the tank is increasing at the moment when the water is 2 feet deep?

(A) 0.5 ft per hr

(B) 1.0 ft per hr

(C) 1.5 ft per hr

(D) 2.0 ft per hr

(E) 2.5 ft per hr

$V = \frac{\pi h^2}{3}(18-h)$

$= 6\pi h^2 - \frac{\pi}{3}h^3$

$\frac{dV}{dt} = 12\pi h \frac{dh}{dt} - \pi h^2 \frac{dh}{dt}$

$h = 2$

$30\pi = 12\pi(2)\frac{dh}{dt} - \pi(4)\frac{dh}{dt}$

$30\pi = 24\pi\frac{dh}{dt} - 4\pi\frac{dh}{dt}$

$30\pi = 20\pi\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{3}{2}$

Ans

28. The graph of the function $f(x) = 2x^{5/3} - 5x^{2/3}$ is increasing on which of the following intervals.

I. $1 < x$ II. $0 < x < 1$ III. $x < 0$

(A) I only (B) II only (C) III only (D) I and II only (E) I and III only

$\frac{10}{3}x^{2/3} - \frac{10}{3}x^{-1/3} = 0$

$\frac{10}{3}x^{-1/3}[x-1] = 0$

$x = 1$

$\frac{10}{3x^{1/3}}(x-1) = 0$

$+ \quad + \quad -$

$0 \quad 1$

$x = 0,$

$\frac{10}{-3}[-2] = +$

$\frac{10}{3}$

$x = 1$

Ans

EXAM I
CALCULUS AB
SECTION I PART B
MULTIPLE-CHOICE
CALCULATORS
Time–50 minutes
Number of questions–17

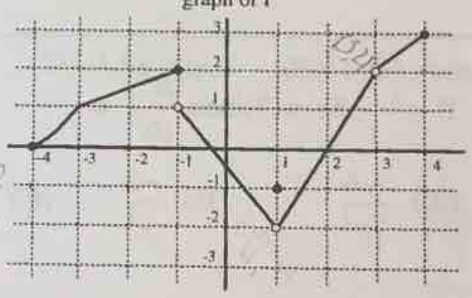
Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS IN THIS PART OF THE EXAMINATION.

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

1. The function $f$ is defined on the interval $[-5, 5]$ and its graph is shown to the right. Which of the following statements are true?




I. $\lim_{x\to 1} f(x) = -1$ No

The II. $\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = 2$

The III. $\lim_{x\to -1^+} f(x) = f(-3)$

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, III

Ans

2. For $f(x) = \sin^2 x$ and $g(x) = 0.5x^2$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, the instantaneous rate of change of $f$ is greater than the instantaneous rate of change of $g$ for which value of $x$?

(A) $-0.8$ (B) 0 (C) 0.9 (D) 1.2 (E) 1.5

$f'(x) = 2\sin x\cos x$
$g'(x) = x$

$2\sin x\cos x > x$
$2\sin x\cos x - x > 0$
$2\sin x\cos x - x = 0$
$x = 0.947$

$2\sin x\cos x > x$
$2\sin x\cos x - x > 0$
$x =$

$-\frac{\pi}{2}$ $-0.947$ $0.947$ $\frac{\pi}{2}$

Ans

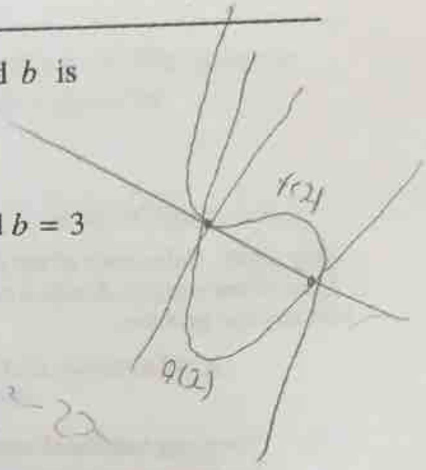
3. If $f(x) = 2x^2 - x^3$ and $g(x) = x^2 - 2x$, for what values of $a$ and $b$ is

$x^2(2-x)$ $\quad$ $x(x-2)$

$$\int_a^b f(x)\,dx > \int_a^b g(x)\,dx\ ?$$

I. $a = -1$ and $b = 0$ II. $a = 0$ and $b = 2$ III. $a = 2$ and $b = 3$

(A) I only
(B) II only
(C) I and II only
(D) I and III only
(E) I, II, III




$\int_a^b 2x^2 - x^3 > \int_a^b x^2 - 2x$

$\left[\frac{2}{3}x^3 - \frac{x^4}{4}\right] > \left[\frac{x^3}{3} - x^2\right]$

Ans

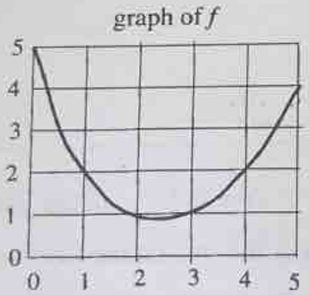
4. If $y^2 - 3x = 7$, then $\dfrac{d^2y}{dx^2} =$

(A) $\dfrac{-6}{7y^3}$ (B) $\dfrac{-3}{y^3}$ (C) 3 (D) $\dfrac{3}{2y}$ (E) $\dfrac{-9}{4y^3}$

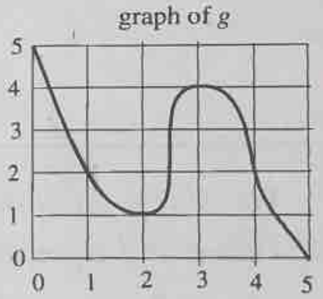
Ans

5. The graphs of functions $f$ and $g$ are shown at the right.
If $h(x) = g[f(x)]$, which of the following statements are true about the function $h$?

graph of $f$




graph of $g$




I. $h(0) = 4$. no
II. $h$ is increasing at $x = 2$. no
III. The graph of $h$ has a horizontal tangent at $x = 4$.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, III

$h(4) = g f(4)$
$= g(2)$
$g'(2) = 0$ ∴ horiz. tangent

$h'(2) > 0$
$h'(x) = g'[f(x)]\, f'(x)$
$h'(2) = g'[f(2)]\, f'(2)$
$= g'(1)\, f'(2)$
$= (-ve)(-ve) = +ve$ ∴ inc.

Ans

$\sqrt{(x-0)^2+(y-0)}$

6. The minimum distance from the origin to the curve $y = e^x$ is

(A) 0.72 (B) 0.74 (C) 0.76 (D) 0.78 (E) 0.80

$(0,0)$ $(0,0)$ $(x, e^x)$

$y = e^{2x} + x^2$
$y' = 2e^{2x} + 2x$
$2e^{2x} + 2x = 0$
$2e^{2x} = -2x$
$x = -0.426302751$

$f(x) = \sqrt{(e^x)^2 + (x)^2}$
$= \sqrt{e^{2x} + x^2}$
$f(-0.426) =$

$D = \sqrt{x^2 + e^{2x}}$

Ans

7. The area of the first quadrant region bounded by the $y$-axis, the line $y = 4 - x$ and the graph of $y = x - \cos x$ is approximately

(A) 4.50 units$^2$ (B) 4.54 units$^2$ (C) 4.56 units$^2$ (D) 4.58 units$^2$ (E) 5.00 units$^2$

Ans

8. The number of inflection points for the graph of $y = 2x + \cos(x^2)$ in the interval $0 \le x \le 5$ is

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

$y = 2x + \cos(x^2)$
$y' = 2 - 2x\sin(x^2)$
$y'' = -2\sin(x^2) - 2x(2x)\cos x^2$
$= -2\sin x^2 - 4x^2\cos x^2 = 0$

$4x^2\cos x^2 = -2\sin x^2$
$-2x^2\cos x^2 = \sin x^2$

Ans

9. The rate at which ice is melting in a pond is given by $\frac{dV}{dt} = \sqrt{1+2^t}$, where $V$ is the volume of ice in cubic feet, and $t$ is the time in minutes. What amount of ice has melted in the first 5 minutes?

(A) 14.49 $ft^3$ (B) 14.51 $ft^3$ (C) 14.53 $ft^3$ (D) 14.55 $ft^3$ (E) 14.57 $ft^3$

$\int_0^5 \sqrt{1+2^t} = 14.530$

Ans

10. The region shaded in the figure at the right is rotated about the $x$-axis. Using the Trapezoid Rule with 5 equal subdivisions, the approximate volume of the resulting solid is

$\frac{6-1}{5} = 1$

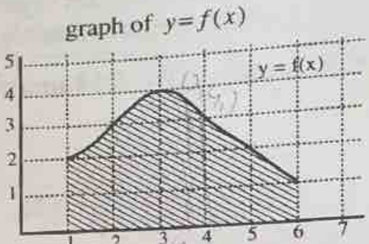
(A) 23 $units^3$

(B) 47 $units^3$

(C) 127 $units^3$

(D) 254 $units^3$

(E) 400 $units^3$




Volume $= \pi \int_1^6 [f(x)]^2 dx$

$= \pi \left[ \frac{6-1}{10} [f(1) + 2[f(2)+f(3)+f(4)+f(5)] + f(6)] \right]$

$= \pi \left[ \frac{1}{2} \right.$

Ans

11. A particle moves along the $x$-axis so that at time $t \geq 0$, its position is given by $x(t) = (t+1)(t-3)^3$. For what values of $t$ is the velocity of the particle increasing?

(A) all $t$ (B) $0 < t < 1$ (C) $0 < t < 3$ (D) $1 < t < 3$ (E) $t < 1$ or $t > 3$

$x(t) = (t+1)(t-3)^3$

$v(t) = (t+1)3(t-3)^2 + (t-3)^3$

$a(t) = 6(t-3)(t+1) + 3(t-3)^2 + 3(t-3)^2$

$6(t-3)(t+1) + 6(t-3)^2 = 0$

$6(t-3)[(t+1)+(t-3)]$

$=(6t-18)(2t-2) = 0$

$t = 3, 1$

+ − +

0 1 3

Ans

12. Let $f(x) = \dfrac{\ln e^{2x}}{x-1}$ for $x > 1$. If $g$ is the inverse of $f$, then $g'(3) =$

(A) 2 (B) 1 (C) 0 (D) $-1$ (E) $-2$

$f(x) = \dfrac{\ln e^{2x}}{x-1},\ x>1$

$x = \dfrac{\ln e^{2y}}{y-1}$

$xy - x = \ln e^{2y}$

$2y = xy - x$

$2y - xy = -x$

$y(2-x) = -x$

$y = \dfrac{-x}{2-x}$

$y = \dfrac{-x}{2-x}$

$g\ y' = \dfrac{(2-x)(-1) + x(-1)}{(2-x)^2}$

$= \dfrac{-2}{(2-x)^2}$

$g'(3) = \dfrac{-2}{(-1)^2} = -2$

$-2(x-2)$

Ans

13. $\displaystyle\int \frac{e^{x^2} - 2x}{e^{x^2}}\,dx$

$\displaystyle\int \frac{e^{x^2} - 2x}{e^{x^2}}$

$= \displaystyle\int 1 - \frac{2x}{e^{x^2}}$

$= x + e^{-x^2}$

$-2x\,e^{-x^2}$

(A) $x - e^{x^2} + C$

(B) $x - e^{-x^2} + C$

(C) $x + e^{-x^2} + C$

(D) $-e^{x^2} + C$

(E) $e^{-x^2} + C$

$-\dfrac{2x}{e^{x^2}}$

Ans

14. How many critical points does the function $f(x) = (x + 2)^5(x^2 - 1)^4$ have?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 9

$f'(x) = 5(x+2)^4(x^2-1)^4 + (x+2)^5(4(x^2-1)^3(2x))$
$= 5(x+2)^4(x^2-1)^4 + 8x(x+2)^5(x^2-1)^3$
$= (x+2)^4(x^2-1)^3[5(x^2-1)+8x(x+2)]$
$= (x+2)^4(x^2-1)^3[13x^2+16x-5]$
$= x=-2,\ x=\pm 1,$

Ans

15. Let $m$ and $b$ be real numbers and let the function $f$ be defined by

$$f(x) = \begin{cases} 1 + 3bx + 2x^2 & \text{for } x \le 1 \\ mx + b & \text{for } x > 1 \ . \end{cases}$$

If $f$ is both continuous and differentiable at $x = 1$, then

(A) $m = 1,\ b = 1$
(B) $m = 1,\ b = -1$
(C) $m = -1,\ b = 1$
(D) $m = -1,\ b = -1$
(E) none of the above

$\lim_{x\to 1^+} mx+b = \lim_{x\to 1^-} 1+3bx+2x^2$
$m+b = 1+3b+2$
$2b-m=-3$

$\lim_{x\to 1^+} \frac{mx-2b-3}{x-1}$ $\boxed{2b-m=-3}$

$\lim_{x\to 1^+}\left[\frac{mx+b-(1+3b+2)}{x-1}\right] = \lim_{x\to 1^-}\left[\frac{1+3bx+2x^2-(1+3b+2)}{x-1}\right]$
$= \cancel{\lim_{x\to 1^+}\left[\frac{mx-3b-3}{x-1}\right]} = \lim_{x\to 1^-}\left[\frac{3b(x-1)+2x^2-2}{x-1}\right]$
$= 3b + 2(x+1)$
$= 3b$

Ans



$2(x+1)(x-1)$

16. Suppose a car is moving with increasing speed according to the following table.

| time (sec) | 0 | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|
| speed (ft/sec) | 30 | 36 | 40 | 48 | 54 | 60 |

The closest approximation of the distance traveled in the first 10 seconds is

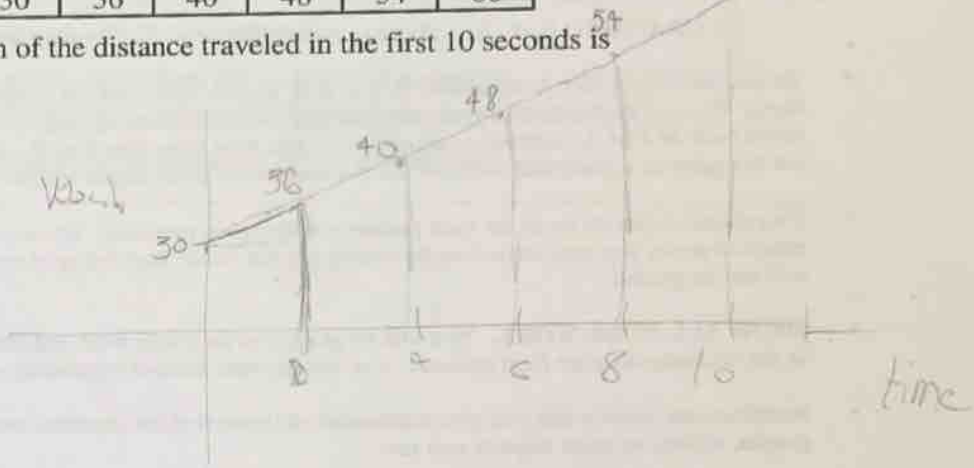
(A) 150 ft

(B) 250 ft

(C) 350 ft

(D) 450 ft

(E) 550 ft




$\frac{1}{2}(30+36)(2)+\frac{1}{2}(36+40)(2)+\frac{1}{2}(40+48)\cdot 2+\frac{1}{2}(48+54)\cdot 2$
$+\frac{1}{2}(54+60)\,2$

Ans $\boxed{\phantom{x}}$

17. Consider the function $F$ defined so that $F(x)+5=\int_2^x \sin\left(\frac{\pi t}{4}\right)dt$.

The value of $F(2)+F'(2)$ is

(A) 0 $\quad -5+1=-4$

(B) 1

(C) $\frac{\pi}{4}$

(D) 4

(E) $-4$

$F(2)=\int_2^2 = -5$

$F'(x) = \frac{d}{dx}\left[\int_0^x \left(\frac{\sin \pi t}{4}\right) - 5\right]$

$= \frac{\sin \pi x}{4}$

$F'(2) = \sin\frac{\pi}{2} = 1$

$F(2) = \int_2^2 \sin\left(\frac{\pi t}{4}\right) - 5$

$= -5$

Ans $\boxed{\phantom{x}}$

EXAM I
CALCULUS AB
SECTION II, PART A
Time–45 minutes
Number of questions–3

A GRAPHING CALCULATOR IS REQUIRED ON THIS PART OF THE EXAMINATION.

- Before you begin Part A of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems and you will be able to come back to Part A (without a calculator), if you have time after Part B. All problems are given equal weight, but the parts of a particular solution are not necessarily given equal weight.

- You should write all work for each problem in the space provided. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed out work will not be graded.

- SHOW ALL YOUR WORK. You will be graded on the correctness and completeness of your methods as well as the accuracy of your final answers. Correct answers without supporting work may not receive full credit.

- Justifications require that you give mathematical (noncalculator) reasons and that you clearly identify functions, graphs, tables, or other objects you use.

- You are permitted to use your calculator in Part A to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate in your exam booklet the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

- Your work must be expressed in mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt($X^2$, $X$, 1, 5).

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

- Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

THE EXAM BEGINS ON THE NEXT PAGE

PLEASE TURN OVER

1. Let $f$ be the function given by $f(t) = 2\pi t + \sin(2\pi t)$.

   (a) Find the value of $t$ in the open interval $(0, 2)$ for which the line tangent at $(t, f(t))$ is parallel to the line through $(0, 0)$ and $(2, 4\pi)$.

   (b) Suppose the given function describes the position of a particle on the $x$-axis for time $0 \le t \le 2$. What is the average velocity of the particle over that interval? 0 ??

   (c) Determine the velocity and the acceleration of the particle at $t = 1$.

---

1) $f(t) = 2\pi t + \sin(2\pi t)$ Slope $= \frac{4\pi}{2} = 2\pi$

$f'(t) = 2\pi + 2\pi\cos(2\pi t) = 2\pi$

$2\pi\cos(2\pi t) = 0$ $0 < x < 2$

$\cos(2\pi t) = 0$ $0 < 2\pi x < 4\pi$

$2\pi t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$ $2\pi + \frac{\pi}{2} =$

$t = \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}$ $3\pi + \frac{\pi}{2} =$

$y = 2\pi t + \sin 2\pi t$

b) $S(t) = 2\pi t + \sin(2\pi t)$

$V(t) = 2\pi + 2\pi\cos(2\pi t)$

$V(t)_{Av} = \frac{1}{2}\int_0^2 2\pi + 2\pi\cos(2\pi t)\,dt$

2. Let $f$ be the function given by $f(x) = x^3 + 3x^2 - x + 2$.

(a) The tangent to the graph of $f$ at the point $P = (-2, 8)$ intersects the graph of $f$ again at the point $Q$. Find the coordinates of the point $Q$.

(b) Find the coordinates of point $R$, the inflection point on the graph of $f$.

(c) Show that the segment $\overline{QR}$ divides the region between the graph of $f$ and its tangent at $P$ into two regions whose areas are in the ratio of $\frac{16}{11}$.

a) $f(x) = x^3 + 3x^2 - x + 2$

$f'(x) = 3x^2 + 6x - 1$  $P(-2, 8)$

$f'(-2) = 3(4) - 12 - 1 = -1$

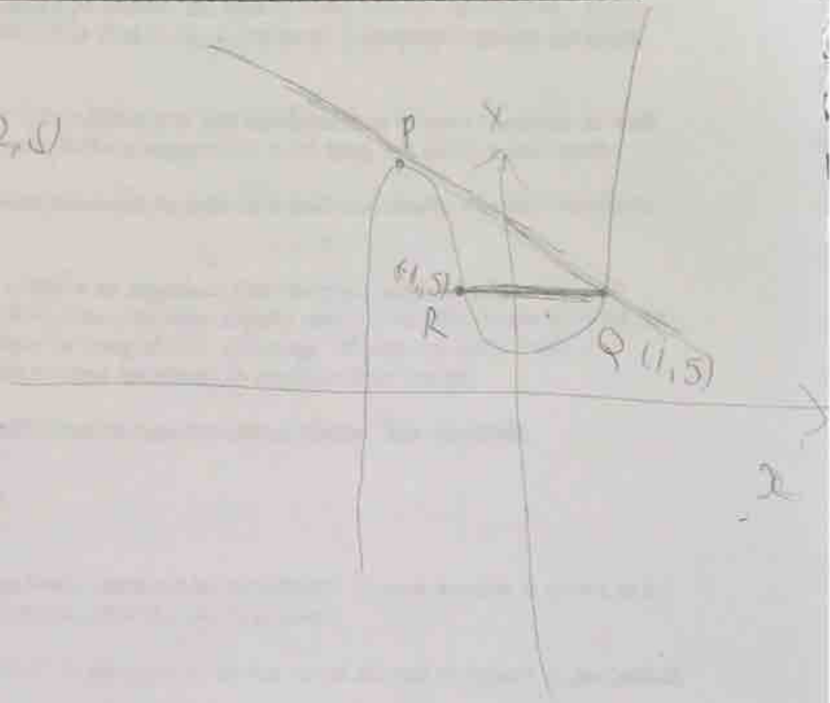
$y - 8 = -1(x + 2)$

$y = -x + 6$

$-x + 6 = x^3 + 3x^2 - x + 2$

$x^3 + 3x^2 - 4 = 0$

$x = 1$,  $(1, 5)$




b) $(-1, 5)$


$\int$

$f'(x) = 3x^2 + 6x - 1$

$f''(x) = 6x + 6 = 0$

$6(x + 1) = 0$

$x = -1$

$f(-1) = -1 + 3 + 1 + 2$

$= 5$

3. Consider the graphs of $y = 3x + c$ and $y^2 = 6x$, where $c$ is a real constant.

   (a) Determine all values of $c$ for which the graphs intersect in two distinct points.

   (b) Suppose $c = -\frac{3}{2}$. Find the area of the region enclosed by the two curves. $\frac{16}{9}$

   (c) Suppose $c = 0$. Find the volume of the solid formed when the region bounded by $y = 3x$ and $y^2 = 6x$ is revolved about the $x$-axis. $\frac{4}{9}\pi$

a) $3x+c=\sqrt{6x}$ $\quad 3x+c=-\sqrt{6x}$

$(3x+c)^2=6x$

$9x^2+6xc+c^2=6x$

$9x^2+6x(c-1)+c^2=0$

$b^2-4ac>0$

$36(c-1)^2-4(9)(c^2)>0$

$36(c^2-2c+1)-36c^2>0$

$-72c+36>0$

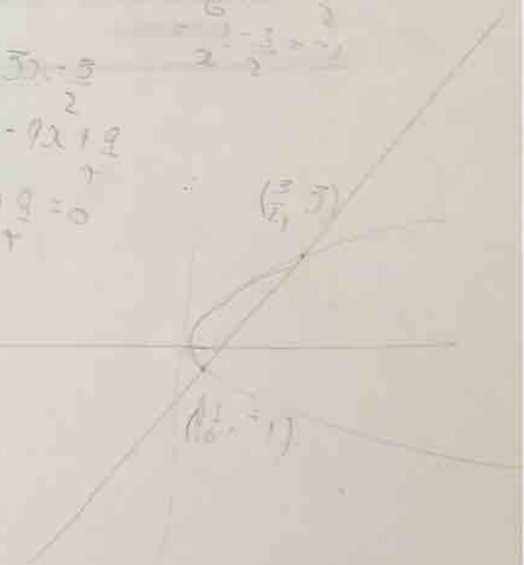
$c<\frac{-36}{-72}$

$c<\frac{1}{2}$

$-\sqrt{6x}=3x-\frac{3}{2}$

$6x=9x^2-9x+\frac{9}{4}$

$9x^2-15x+\frac{9}{4}=0$




b) $3x-\frac{3}{2}=\sqrt{6x}$

Area $=\int_0^{1.5}$

A CALCULATOR MAY **NOT** BE USED ON THIS PART OF THE EXAMINATION.
DURING THIS 45 MINUTES YOU MAY GO BACK AND WORK ON PART A.

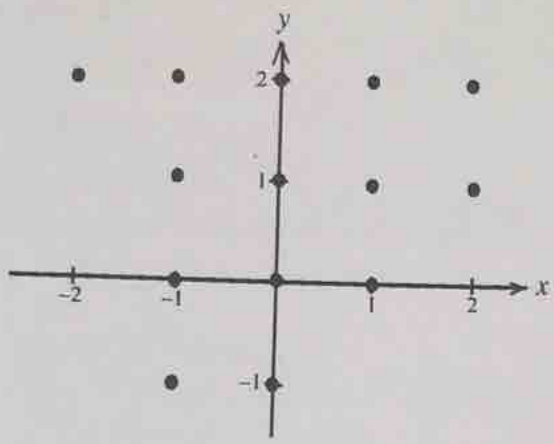
4. Consider the differential equation $\frac{dy}{dx} = x - y$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the fourteen points indicated.

   (b) Sketch the solution curve that contains the point $(-1, 1)$.

   (c) Find an equation for the straight line solution through the point $(1, 0)$.

   (d) Show that if $C$ is a constant, then $y = x - 1 + Ce^{-x}$ is a solution of the differential equation.




$y = x - 1$ ∴

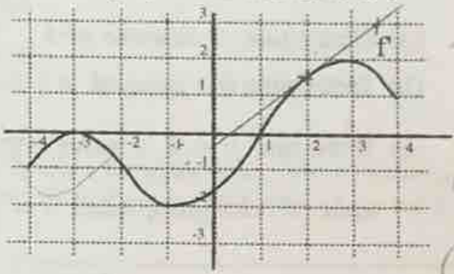
$$\boxed{\frac{dy}{dx} = x - y}$$

$dy = (x - y)dx$

Separate Variables

5. The graph of the **derivative** of $f$ is shown in the figure.

the graph of the derivative of $f$




$f(3) = 1$

(a) Suppose that $f(3) = 1$. Find an equation of the line tangent to the graph of $f$ at the point (3, 1).

(b) Where does $f$ have a local minimum? Explain briefly. $x = 1$

$(3\frac{1}{2}, ?)$

(c) Estimate $f''(2)$. 0.75 ???

$(2, \frac{3}{2})$

(d) Where does $f$ have an inflection point? Explain briefly.

(e) Where does $f$ achieve its maximum on the interval [1, 4]? $x = 4$

$m = \frac{3/2}{3/2} = 1$

a) $y - 1 = 2(x - 3)$

$y = 2x - 5$ .

b) $x = 1$

SLOPE - CURVE

c. $f''(2) = \frac{f(3) - f(1)}{3 - 1} = \frac{2 - 0}{2} = 1$

$\frac{x}{2} = \frac{7}{4}$

$4x = c$

$x = \frac{5}{2}$

No. rel max

e) min a

no relative max

∴ max must occur at either endpoint

$\int_{-4}^{4} f'(x) < 0$

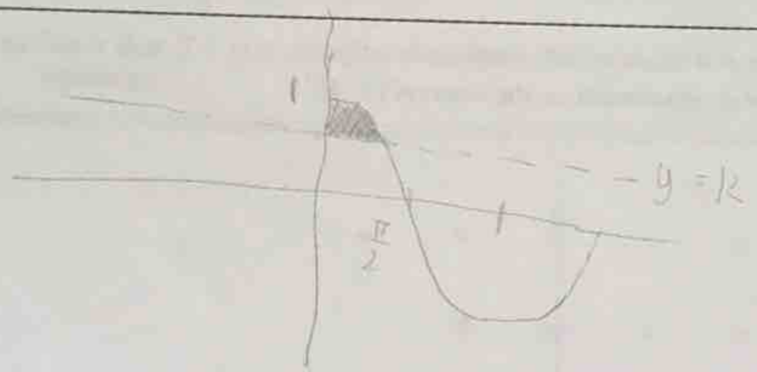
$\boxed{\begin{array}{l} f(4) - f(-4) < 0 \\ f(4) < f(-4) \end{array}}$

6. Let $A$ be the area of the region in the first quadrant under the graph of $y = \cos x$ and above the line $y = k$ for $0 \le k \le 1$.

(a) Determine $A$ in terms of $k$.

(b) Determine the value of $A$ when $k = \frac{1}{2}$.

(c) If the line $y = k$ is moving upward at the rate of $\frac{1}{\pi}$ units per minute, at what rate is the area, $A$, changing when $k = \frac{1}{2}$?




$\cos x = 1/a$

$x = \cos^{-1} k$

$A = \int_0^{\cos^{-1} k} [\cos x - k]$

$= \cos(\cos^{-1} k - k - (\cos 0 - 0)$

$= k - k - 1$

$= -1$

$\frac{dy}{dt} = \frac{1}{\pi}$

$\frac{dA}{dt} = \frac{1}{3}$ (n²)²/4

EXAM II
CALCULUS AB
SECTION I PART A
MULTIPLE CHOICE
NO CALCULATORS
Time–55 minutes
Number of questions–28

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value. Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

$4(8)-3(2)-1$

1. Let $f(x) = 4x^3 - 3x - 1$. An equation of the line tangent to $y = f(x)$ at $x = 2$ is

(A) $y = 25x - 5$

(B) $y = 45x + 65$

(C) $y = 45x - 65$

(D) $y = 65 - 45x$

(E) $y = 65x - 45$

$y = 4x^3 - 3x - 1$ $(2, 25)$
$y' = 12x^2 - 3$
$y'(2) = 12(4) - 3 = 45$
$y - 25 = 45(x-2)$
$y = 45x - 90 + 25$
$y = 45x - 65$

Ans

2. $\int_0^1 \sin \pi x \, dx =$ $\left[\frac{-\cos \pi x}{\pi}\right]_0^1 = \frac{-\cos \pi}{\pi} + \frac{\cos 0}{\pi} = \frac{-(-1)}{\pi} + \frac{1}{\pi} = \frac{2}{\pi}$

(A) $\frac{2}{\pi}$ (B) $\frac{1}{\pi}$ (C) $0$ (D) $-\frac{2}{\pi}$ (E) $-\frac{1}{\pi}$

Ans

3. $\lim\limits_{h\to 0}\left(\dfrac{\cos(x+h)-\cos x}{h}\right)=$

$f(x) = \cos x$

$\lim\limits_{h\to 0} f(x) = \cos 0 = 1$

$f(x) = \cos x$

$f'(x) = -\sin x$

$\dfrac{f(x+h)-f(x)}{h}$

(A) $\sin x$

(B) $-\sin x$

(C) $\cos x$

(D) $-\cos x$

(E) does not exist

Ans

4. On which of the following intervals, is the graph of the curve $y = x^5 - 5x^4 + 10x + 15$ concave up ?

I. $x < 0$ II. $0 < x < 3$ III. $x > 3$

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) II and III only

Ans

5. The region bounded by the $x$-axis and the part of the graph of $y = \sin x$ between $x = 0$ and $x = \pi$ is separated into two regions by the line $x = k$. If the area of the region for $0 \le x \le k$ is one-third the area of the region for $k \le x \le \pi$, then $k =$

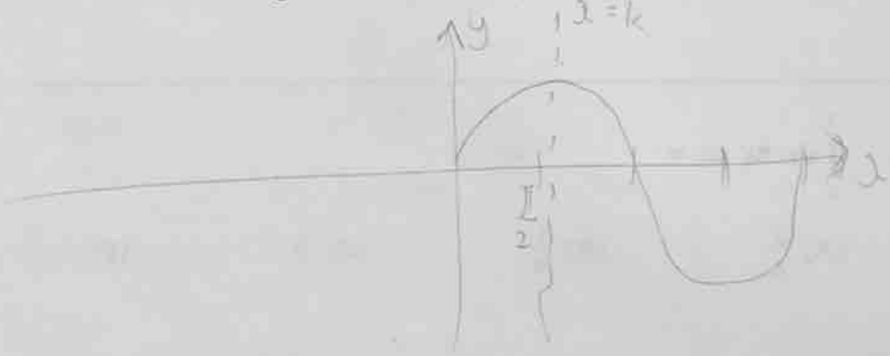
(A) $\arcsin\frac{1}{3}$

(B) $\arcsin\frac{1}{4}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{3}$

(E) $\frac{\pi}{4}$




$\int_0^k \sin x = \frac{1}{3}\int_k^{\pi} \sin x$

Ans

6. A particle starts at time $t = 0$ and moves along a number line so that its position, at time $t \geq 0$, is given by $x(t) = (t-2)^3(t-6)$. The particle is moving to the right for

(A) $0 < t < 5$

(B) $2 < t < 6$

(C) $t > 5$

(D) $t \geq 0$

(E) never

$x(t) = (t-2)^3(t-6)$
$v(t) = 3(t-2)^2(t-6) + (t-2)^3 = 0$
$= (t-2)^2[3(t-6)+(t-2)]$
$= (t-2)^2[4t-20]$
$t = 2,\ t = 5$

$-$ $-$ $+$
0 2 5

Ans

---

??? 7. If $\dfrac{dy}{dx} = \sec x$, then $y =$

(A) $\ln|\cos x| + C$

(B) $\sec x + \tan x + C$

(C) $\ln|(\sec x)(\tan x)| + C$

(D) $(\sec x)(\tan x) + C$

(E) $\ln|\sec x + \tan x| + C$

$y = \int \sec x\, dx$
$= \int \frac{1}{\cos x}\, dx$

Ans

---

8. $\displaystyle\int_{\pi/4}^{\pi/3} \frac{\sec^2 x}{\tan x}\, dx =$

(A) $\ln\sqrt{3}$ (B) $-\ln\sqrt{3}$ (C) $\ln\sqrt{2}$ (D) $\sqrt{3}-1$ (E) $\ln\frac{\pi}{3} - \ln\frac{\pi}{4}$

$\int \frac{\sec^2 x}{\tan x}$ $\left[\ln|\tan x|\right]_{\pi/4}^{\pi/3}$

$\ln \tan \frac{\pi}{3} - \ln \tan \frac{\pi}{4}$
$= \ln\sqrt{3} - \ln 1$
$= \ln\sqrt{3}$


$\tan 60 = \frac{\sqrt{3}}{1}$

Ans

9. What is $\lim_{x \to \infty} \frac{x^2 - 6}{2 + x - 3x^2}$ ?

(A) $-3$ (B) $-\frac{1}{3}$ (C) $\frac{1}{3}$ (D) 2 (E) The limit does not exist.

Ans

???

10. $\int_0^2 \sqrt{x^2 - 4x + 4}\, dx$ is:

(A) 1

(B) $-1$

(C) $-2$

(D) 2

(E) None of the above

$\sqrt{x^2-4x+4}$

$(x-2)^{2\cdot\frac{1}{2}}$

$\sqrt{(x-2)^2}$

$= (x-2) = \left[\frac{x^2}{2} - 2x\right]_0^2$

$= 2 - 4 = -2$

Ans

11. If $g(x) = \frac{x-2}{x+2}$, then $g'(2) =$

(A) 1

(B) $-1$

(C) $\frac{1}{4}$

(D) $-\frac{1}{4}$

(E) 0

Ans

12. If $\frac{dy}{dx} = 2xy$ and if $y = 4$ when $x = 0$, then $y =$

(A) $e^{x^2}$

(B) $4e^{x^2}$

(C) $4 + e^{x^2}$

(D) $4 + 4e^{x^2}$

(E) $4 + 2e^{x^2}$

$\frac{dy}{dx} = 2xy$

$\int \frac{dy}{y} = \int 2x\,dx$

$\ln y = x^2 + c$

$\ln 4 = c$

$\ln y = x^2 + \ln 4$

$y = e^{x^2 + \ln 4}$

$= e^{x^2} \cdot e^{\ln 4} = 4e^{x^2}$

Ans

13. The fourth derivative of $f(x) = (2x - 3)^4$ is

(A) $24(2^4)$

(B) $24(2^3)$

(C) $24(2x - 3)$

(D) $24(2^5)$

(E) 0

Ans

14. If $\int_2^4 f(x)\,dx = 6$, then $\int_2^4 (f(x) + 3)\,dx =$

(A) 3

(B) 6

(C) 9

(D) 12

(E) 15

$\int_2^4 f(x) + \int_2^4 3$

$= 6 + [3x]_2^4$

$= 6 + 12 - 6$

$= 12$

Ans

15. If $\tan(x + y) = x$, then $\dfrac{dy}{dx} =$

(A) $\tan^2(x + y)$

(B) $\sec^2(x + y)$

(C) $\ln|\sec(x + y)|$

(D) $\sin^2(x + y) - 1$

(E) $\cos^2(x + y) - 1$

$\tan(x+y) = x$
$(1+y')\sec^2(x+y) = 1$
$\sec^2(x+y) + y'\sec^2(x+y) = 1$
$y' = \dfrac{1-\sec^2(x+y)}{\sec^2(x+y)}$

Ans $\square$

16. If $f(x) = e^{2x}$ and $g(x) = \ln x$, then the derivative of $y = f(g(x))$ at $x = e$ is

(A) $e^2$

(B) $2e^2$

(C) $2e$

(D) $2$

(E) undefined

$y = e^{2\ln x}$
$y' = \dfrac{2}{x} e^{2\ln x}$
$y'(e) = \dfrac{2}{e} e^{2\ln e}$
$= \dfrac{2}{e} e^2$
$= 2e^{2-1} = 2e$

Ans $\square$

17. The area of the region bounded by the lines $x = 1$ and $y = 0$ and the curve $y = xe^{x^2}$ is

(A) $1 - e$ units$^2$

(B) $e - 1$ units$^2$

(C) $\dfrac{e-1}{2}$ units$^2$

(D) $\dfrac{1-e}{2}$ units$^2$

(E) $\dfrac{e}{2}$ units$^2$

$\int_0^1 xe^{x^2}$
$= \left[\dfrac{e^{x^2}}{2}\right]_0^1$
$= \dfrac{e}{2} - \dfrac{1}{2}$

Ans $\square$

18. If $h(x) = (x^2 - 4)^{3/4} + 1$, then the value of $h'(2)$ is

(A) 3

(B) 2

(C) 1

(D) 0

(E) does not exist

Ans $\square$

19. The derivative of $\sqrt{x} - \dfrac{1}{x\sqrt[3]{x}}$ is

$x^{1/2} - \dfrac{1}{x^{4/3}}$ $\quad x \cdot x^{1/3}$

$\frac{1}{2}x^{-1/2} + \frac{4}{3}x^{-7/3}$ $\quad x^{-4/3}$ $\quad -\frac{4}{3} + \frac{3}{3} = ...$

(A) $\frac{1}{2}x^{-1/2} - x^{-4/3}$

(B) $\frac{1}{2}x^{-1/2} + \frac{4}{3}x^{-7/3}$

(C) $\frac{1}{2}x^{-1/2} - \frac{4}{3}x^{-1/3}$

(D) $-\frac{1}{2}x^{-1/2} + \frac{4}{3}x^{-7/3}$

(E) $-\frac{1}{2}x^{-1/2} - \frac{4}{3}x^{-1/3}$

Ans $\square$

20. The function $f$ is continuous at $x = 1$.

If $f(x) = \begin{cases} \dfrac{\sqrt{x+3} - \sqrt{3x+1}}{x-1} & \text{for } x \neq 1 \\ k & \text{for } x = 1 \end{cases}$ then $k =$

$\sqrt{x+3} + \sqrt{3x+1}$

$\dfrac{x+3-(3x+1)}{x-1(\sqrt{x+3}+\sqrt{3x+1})}$

$\dfrac{-2x+2}{x-1} = \dfrac{-(x-1)}{1}$

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$ (E) none of the above

$\lim_{x \to 1}$

$= -\dfrac{1}{\sqrt{x+3}+\sqrt{3x+1}} = \dfrac{-1}{2+2} = -\dfrac{1}{4}$

Ans $\square$

21. An equation of the normal to the graph of $f(x) = \dfrac{x}{2x-3}$ at $(1, f(1))$ is

(A) $3x + y = 4$

(B) $3x + y = 2$

(C) $x - 3y = -2$

(D) $x - 3y = 4$

(E) $x + 3y = 2$

Ans $\square$

22. Let $f(x) = x \ln x$. The minimum value attained by $f$ is

(A) $-\dfrac{1}{e}$

(B) 0

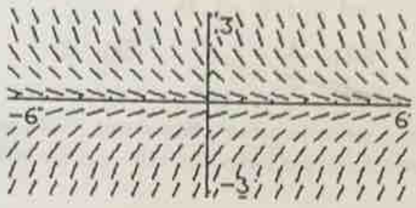
(C) $\dfrac{1}{e}$

(D) $-1$

(E) There is no minimum.

Ans $\square$

23. The slope field for a differential equation $\frac{dy}{dx} = f(x, y)$ is given in the figure. The slope field corresponds to which of the following differential equations?




(A) $\frac{dy}{dx} = x + y$

(B) $\frac{dy}{dx} = y^2$

(C) $\frac{dy}{dx} = -y$

(D) $\frac{dy}{dx} = e^{-x}$

(E) $\frac{dy}{dx} = 1 - \ln x$

Ans

24. The average value of $\sec^2 x$ over the interval $0 \le x \le \frac{\pi}{4}$ is

(A) $\frac{\pi}{4}$ (B) $\frac{4}{\pi}$ (C) $\frac{\pi}{8}$ (D) 1 (E) none of the above

$\frac{1}{\frac{\pi}{4} - 0} \int_0^{\pi/4} \sec^2 x \, dx$

$= \frac{4}{\pi} [\tan x]_0^{\pi/4}$

$= \frac{4}{\pi} [\tan \frac{\pi}{4}] = \frac{4}{\pi}$

Ans

25. Suppose that $g$ is a function that is defined for all real numbers. Which of the following conditions assures that $g$ has an inverse function?

(A) $g'(x) < 1$, for all $x$

(B) $g'(x) > 1$, for all $x$

(C) $g''(x) > 0$, for all $x$

(D) $g''(x) < 0$, for all $x$

(E) $g$ is continuous.

Ans

26. The average rate of change of the function $f(x) = \left|x^2 - 2|x+2|\right|$ over the interval $-3 < x < -1$ is

(A) $-3$ (B) $-2$ (C) $-1$ (D) $1$ (E) $2$

$\frac{f(-1) - f(-3)}{-1+3} = \frac{1-7}{2} = \frac{-6}{2} = -3$

Ans $\square$

27. If the function $G$ is defined for all real numbers by $G(x) = \int_0^{2x} \cos(t^2)\, dt$, then $G'(\sqrt{\pi}) =$

(A) 2 (B) 1 (C) 0 (D) $-1$ (E) $-2$

$G'(x) = \cos x^2 \cdot 2$
$= 2\cos x^2$
$= 2\cos \pi$
$= 2(-1) = -2$

$2\cos 4x = G'(x)$

$G'(\sqrt{\pi}) = 2\cos 4\sqrt{\pi}$
$=$

Ans $\square$

28. At time $t$ a particle moving along the $x$-axis is at position $x$. The relationship between $x$ and $t$ is given by: $tx = x^2 + 8$. At $x = 2$ the velocity of the particle is

(A) 1

(B) 2

(C) 6

(D) $-2$

(E) $-1$

Ans $\square$

EXAM II
CALCULUS AB
SECTION I PART B
MULTIPLE-CHOICE
CALCULATORS
Time–50 minutes
Number of questions–17

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS IN THIS PART OF THE EXAMINATION.

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

1. Which of the following functions have a derivative at $x = 0$?

I. $y = |x^3 - 3x^2|$ yes

II. $y = \sqrt{x^2 + .01} - |x - 1|$

III. $y = \dfrac{e^x}{\cos x}$ $-(-x+1)$

$x^3 - 3x^2 \ge 0$
$x^2(x-3) \ge 0$
$x = 0, x = 3$

− − +
0 3

$y = \begin{cases} x^3 - 3x^2, & \text{for } x \ge 3, x = 0 \\ 3x^2 - x^3 & \text{for } x < 3, x \ne 0 \end{cases}$

(A) None (B) II only (C) III only (D) II and III only (E) I, II, III

$y = \begin{cases} \sqrt{x^2 + 0.01} - (x-1) & \text{for } x \ge 1 \\ \sqrt{x^2 + 0.01} + (x-1) & \text{for } x < 1 \end{cases}$

$y' = 3x^2 - 6x$, $6x - 3x^2$, $x = 0$, $x \ne 0$

Ans

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2. Water is pumped into an empty tank at a rate of $r(t) = 20e^{0.02t}$ gallons per minute. Approximately how many gallons of water have been pumped into the tank in the first five minutes?

$S(t) = \int_0^5 20e^{0.02t}$

(A) 20 gal
(B) 22 gal
(C) 85 gal
(D) 105 gal
(E) 150 gal

Ans

3. Consider the function $f(x) = \dfrac{6x}{a + x^3}$ for which $f'(0) = 3$. The value of $a$ is

(A) 5
(B) 4
(C) 3
(D) 2
(E) 1

$f'(x) = \dfrac{(a+x^3)6 - 6x(3x^2)}{(a+x^3)^2}$

$f'(x) = \dfrac{6a + 6x^3 - 18x^3}{(a+x^3)^2}$

$3 = \dfrac{6a}{a^2}$

$3a^2 - 6a = 0$

$3a(a-2) = 0$

$a = 0,\ a = 2$

Ans

4. Which of the following is true about the function $f$ if $f(x) = \dfrac{(x-1)^2}{2x^2 - 5x + 3}$ ?

I. $f$ is continuous at $x = 1$.
II. The graph of $f$ has a vertical asymptote at $x = 1$.
III. The graph of $f$ has a horizontal asymptote at $y = \frac{1}{2}$.

(A) I only  (B) II only  (C) III only  (D) II and III only  (E) I, II, III

$2x^2 - 2x - 3x + 3$

$2x(x-1) - 3(x-1)$

$(2x-3)(x-1)$

$\dfrac{(x-1)^2}{(2x-3)(x-1)}$

$\dfrac{x-1}{2x-3}$ $x \neq 1$

$\lim\limits_{x \to 1^+} \dfrac{(x-1)}{2x-3} = \dfrac{0}{-1} = 0$

$\lim\limits_{x \to 1^-} = 0$

$\dfrac{(x-1)^2}{(x-1)(2x-3)} = \dfrac{x-1}{2x-3}$

$\dfrac{x^2 - 2x + 1}{2x^2 - 5x + 3}$

$x \neq 1$

Ans

5. If $y = u + 2e^u$ and $u = 1 + \ln x$, find $\dfrac{dy}{dx}$ when $x = \dfrac{1}{e}$

(A) $e$  (B) $2e$  (C) $3e$  (D) $\dfrac{2}{e}$  (E) $\dfrac{3}{e}$

$\dfrac{dy}{du} = 1 + 2e^u$  $\dfrac{du}{dx} = \dfrac{1}{x}$

$\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx} = \dfrac{1+2e^u}{x} = \dfrac{1+2e^u}{1/e} = e + 2e^{u+1}$

$= e + 2e^1$

$y = 1 + \ln x + 2e^{1+\ln x}$

$y' = \dfrac{1}{x} + 2(1/x)e^{1+\ln x} = \dfrac{1}{x} + \dfrac{2}{x}e^{1+\ln x} = \dfrac{1}{x} + \dfrac{2}{x}e \cdot x$

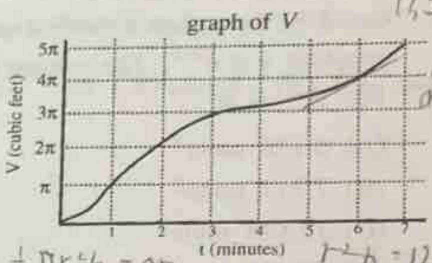
$= \dfrac{1}{x} + 2e = e + 2e = 3e$

Ans

Slope of Curve

(5, 3.5π) (7, 5π)

6. Sand is being dumped on a pile in such a way that it always forms a cone whose base radius is always 3 times its height. The function $V$ whose graph is sketched in the figure gives the volume of the conical sand pile, $V(t)$, measured in cubic feet, after $t$ minutes. $\left(V(t) = \frac{1}{3}\pi r^2 h\right)$ At what approximate rate is the radius of the base changing after 6 minutes?




$\frac{dV}{dt} = \frac{1.5\pi}{2} = \frac{3}{4}\pi$

$r = 3h$ $h = \frac{r}{3}$

$\frac{1}{3}\pi r^2 h = 4\pi$ $r^2 h = 12$ $r^2\left(\frac{r}{3}\right) = 12$ $\frac{r^3}{3} = 12$ $r^3 = 36$ $r = \sqrt[3]{36}$

(A) 0.22 ft/min (B) 0.28 ft/min (C) 0.34 ft/min (D) 0.40 ft/min (E) 0.46 ft/min

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi r^2 \left(\frac{r}{3}\right)$

$V = \frac{1}{9}\pi r^3$

$\frac{dV}{dt} = \frac{1}{3}\pi r^2 \frac{dr}{dt}$

$\frac{3}{4}\pi = \frac{1}{3}\pi (36)^{2/3} \frac{dr}{dt}$

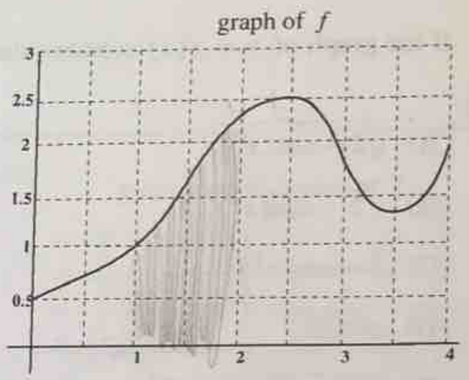
$\frac{9}{4} = (36)^{2/3} \frac{dr}{dt}$

$\frac{dr}{dt} = 0.20637$

$(36^{1/3})^2$

Ans

7. A graph of the function $f$ is shown at the right. Which of the following statements are true?




I. $f(1) > f'(3)$ +ve −ve

II. $\int_1^2 f(x)\,dx > f'(3.5)$ +ve 0

III. $\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} > \frac{f(2.5)-f(2)}{2.5-2}$ $f'(2)$

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, III

Ans

8. Given: $5x^3 + 40 = \int_a^x f(t)\,dt$. The value of $a$ is

(A) −2
(B) 2
(C) 1
(D) −1
(E) 0

$15x^2 = f(x)$

$5x^3 + 40 = \int_a^x 15x^2$

$5x^3 + 40 = \left[5x^3\right]_a^x$

$5x^3 + 40 = 5x^3 - 5a^3$

$-5a^3 = 40$

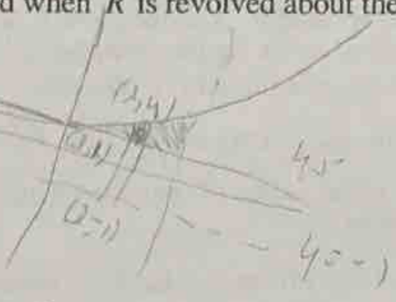
$a^3 = -8$

$a = -2$

Ans

9. Let $R$ be the region in the first quadrant enclosed by the lines $x = \ln 3$ and $y = 1$ and the graph of $y = e^{x/2}$. The volume of the solid generated when $R$ is revolved about the line $y = -1$ is

(A) 5.128 units$^3$
(B) 7.717 units$^3$
(C) 12.845 units$^3$
(D) 15.482 units$^3$
(E) 17.973 units$^3$



$R(x) = e^{x/2}+1$
$r(x) = 1+1 = 2$
volume $= \pi \int_0^{\ln 3} \left((e^{x/2}+1)^2 - 2^2\right) dx$

Ans

---

10. If the graph of $y = f(x)$ contains the point $(0, 1)$, and if $\dfrac{dy}{dx} = \dfrac{x\sin(x^2)}{y}$, then $f(x) =$

(A) $\sqrt{2-\cos(x^2)}$
(B) $\sqrt{2}-\cos(x^2)$
(C) $2-\cos(x^2)$
(D) $\cos(x^2)$
(E) $\sqrt{2-\cos x}$

$\dfrac{dy}{dx} = \dfrac{x\sin(x^2)}{y}$
$\int y\,dy = \int x\sin(x^2)\,dx$
$\dfrac{y^2}{2} = -\dfrac{\cos(x^2)}{2} + c$
at $(0,1)$
$y^2 = -\cos(x^2)+c$
$1 = -\cos 0 + c$
$c = 2$
$y^2 = -\cos x^2 + 2$
$y = \sqrt{2-\cos x^2}$

Ans

---

11. If $y = \sin u$, $u = v - \dfrac{1}{v}$, and $v = \ln x$, then value of $\dfrac{dy}{dx}$ at $x = e$ is

(A) 0
(B) 1
(C) $\dfrac{1}{e}$
(D) $\dfrac{2}{e}$
(E) $\cos e$

$\dfrac{dy}{dv} =$
$y = \sin u$
$y = \sin\left(v - \dfrac{1}{v}\right)$
$y = \sin\left(\ln x - \dfrac{1}{\ln x}\right)$ $\quad \ln x - (\ln x)^{-1}$
$\dfrac{dy}{dx}\left[\dfrac{1}{x} + (\ln x)^{-2}\left(\dfrac{1}{x}\right)\right]\cos\left(\ln x - \dfrac{1}{\ln x}\right)$
$= \left[\dfrac{1}{x} + \dfrac{1}{x(\ln x)^2}\right]\cos\left(\ln x - \dfrac{1}{\ln x}\right)$

Ans

$= \dfrac{1}{e} + \dfrac{e}{e^2} = \dfrac{1+e}{e^2}$ $\quad \left[= \dfrac{1}{e} + \dfrac{1}{e}\right](\cos)(0)$

12. The area of the region bounded by the graphs of $y = 2 - |x - 3|$ and $y = x^2 - 2x$ is

(A) 1.86 units$^2$ (B) 1.88 units$^2$ (C) 1.90 units$^2$ (D) 1.92 units$^2$ (E) 1.94 units$^2$

Calc

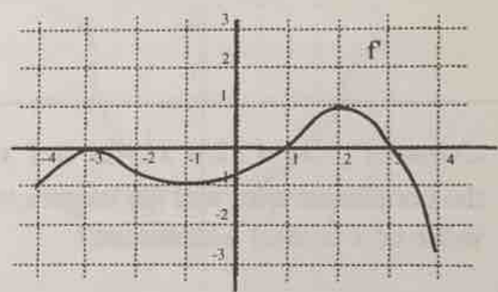
$= \int_{0.382}^{2.618} 2 - |x-3| - (x^2 - 2x)$

(2.618, ...)

Ans

13. The figure shows the graph of $f'$, the *derivative* of a function $f$. The domain of $f$ is the interval $-4 \le x \le 4$. Which of the following are true about the graph of $f$?

graph of the derivative of $f$




I. At the points where $x = -3$ and $x = 2$ there are horizontal tangents. No

II. At the point where $x = 1$ there is a relative minimum point. Yes

III. At the point where $x = -3$ there is an inflection point.

(A) None (B) II only (C) III only (D) II and III only (E) I, II, III

Ans

14. Consider the function $f(x) = (x^2 - 5)^3$ for all real numbers $x$. The number of inflection points for the graph of $f$ is

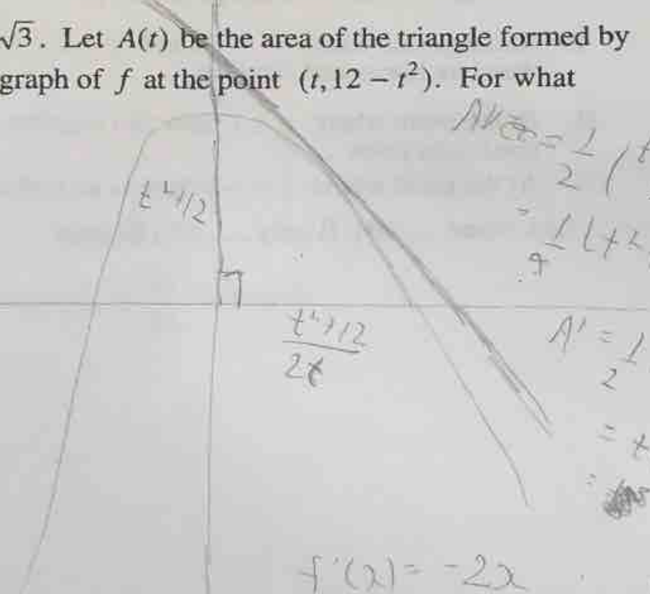
(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

$f(x) = (x^2-5)3$
$f'(x) = 3(x^2-5)^2(2x)$
$= 6x(x^2-5)^2$
$f''(x) = 6(x^2-5)^2 + 6x(2(x^2-5)2x)$
$= 6(x^2-5)^2 + 24x(x^2-5)$
$= 3(x^2-5)[2(x^2-5)+8x]$
$= 3(x^2-5)(2x^2+8x-10)$
$(x^2-5)(x^2+4x-5)$
$(x^2-5)(x-1)(x+5)=0$
$x=\pm\sqrt{5}$, $x=1$, $x=-5$

Ans

15. Consider $f(x) = 12 - x^2$ for $0 \le x \le 2\sqrt{3}$. Let $A(t)$ be the area of the triangle formed by the coordinate axes and the tangent to the graph of $f$ at the point $(t, 12 - t^2)$. For what value of $t$ is $A(t)$ a minimum?

(A) 0
(B) 1
(C) 2
(D) 3
(E) $2\sqrt{3}$




$A(t) = \frac{1}{2}\left(\frac{t^2+12}{2}\right)(t^2+12)$
$= \frac{1}{4}(t^2+12)^2$
$A' = \frac{1}{2}(t^2+12)(2t)$
$= t(t^2+12) = 0$
$t=0$
$t^2=-12$
$t=0$

12

$x=0$; $y=t^2+12$
$y=0$ $t^2-2tx+12=0$

$t^2-2tx+12=0$
$2tx=t^2+12$
$x=\frac{t^2+12}{2t}$

$f'(x)=-2x$
$=-2t$

$y-12+t^2=-2t(x-t)$
$y-12+t^2=-2tx+2t^2$
$y=t^2-2tx+12$

Ans

16. The acceleration of a particle at time $t$ moving along the $x$-axis is given by: $a = 4e^{2t}$. At the instant when $t = 0$, the particle is at the point $x = 2$ moving with velocity $v = -2$. The position of the particle at $t = \frac{1}{2}$ is

(A) $e - 3$ (B) $e - 2$ (C) $e - 1$ (D) $e$ (E) $e + 1$

$a = 4e^{2t}$
$V = \int 4e^{2t}$
$= \frac{4e^{2t}}{2} = 2e^{2t} + c$
$-2 = 2 + c$
$c = -4$
$S = \int 2e^{2t} - 4$
$= e^{2t} - 4t + c$
$2 = 1 - 0 + c$
$c = 1$
$e^{2t} - 4t + 1$
$e - 2 + 1 = e - 1$

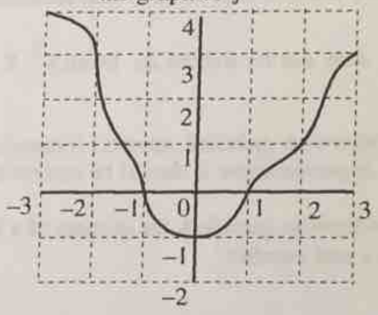
Ans

17. The graph of $f'$, the derivative of a function $f$, is shown at the right.

Which of the following statements are true about the function $f$?

I. $f$ is increasing on the interval $(-2, -1)$. Yes
II. $f$ has an inflection point at $x = 0$. Yes
III. $f$ is concave up on the interval $(-1, 0)$. No

the graph of $f'$




(A) I only (B) II only (C) III only (D) I and II only (E) II and III only

Ans

EXAM II
CALCULUS AB
SECTION II, PART A
Time–45 minutes
Number of questions–3

A GRAPHING CALCULATOR IS REQUIRED ON THIS PART OF THE EXAMINATION.

- Before you begin Part A of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems and you will be able to come back to Part A (without a calculator), if you have time after Part B. All problems are given equal weight, but the <u>parts</u> of a particular solution are not necessarily given equal weight.

- You should write all work for each problem in the space provided. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed out work will not be graded.

- SHOW ALL YOUR WORK. You will be graded on the correctness and completeness of your methods as well as the accuracy of your final answers. Correct answers without supporting work may not receive full credit.

- Justifications require that you give mathematical (noncalculator) reasons and that you clearly identify functions, graphs, tables, or other objects you use.

- You are permitted to use your calculator in Part A to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate in your exam booklet the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

- Your work must be expressed in mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt($X^2$, $X$, 1, 5).

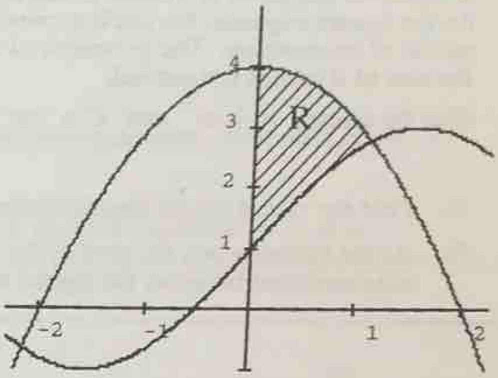
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

- Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

THE EXAM BEGINS ON THE NEXT PAGE

PLEASE TURN OVER

1. Let $R$ be the shaded region in the first quadrant enclosed by the $y$-axis and the graphs of $y = 4 - x^2$ and $y = 1 + 2\sin x$ as shown in the figure at the right.



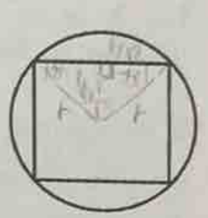

(a) Find the area of $R$.

(b) Find the volume of the solid generated when $R$ is revolved about the $x$-axis.

(c) Find the volume of the solid whose base is $R$ and whose cross sections perpendicular to the $x$-axis are squares.

2. A square is inscribed in a circle as shown in the figure at the right. As the square expands, the circle expands to maintain the four points of intersection. The perimeter of the square is increasing at the rate of 8 inches per second.

(For the circle: $A = \pi r^2$ and $C = 2\pi r$.)



(a) Find the rate at which the circumference of the circle is increasing.

(b) At the instant when the area of the square is 16 square inches, find the rate at which the area enclosed between the square and the circle is increasing.

a) $\frac{dP}{dt} = 8$

$P = 4(2r)$
$= 8r$

$\frac{dP}{dt} = 8\frac{dr}{dt} = 8$

$\frac{dr}{dt} = 1$

$C = 2\pi r$

$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$

$= 2\pi(1)$

b) $\pi r^2 = 16$

$r^2 = \frac{16}{\pi}$

$r = \frac{4}{\sqrt{\pi}}$

$A$ area $= 2\pi r - 4r^2$

$\frac{dA}{dt} = 2\pi \frac{dr}{dt} - 8r\frac{dr}{dt}$

$= 2\pi - 8r$

$= 2\pi - 8\left(\frac{4}{\sqrt{\pi}}\right)$

$= 2\pi - \frac{32}{\sqrt{\pi}}$

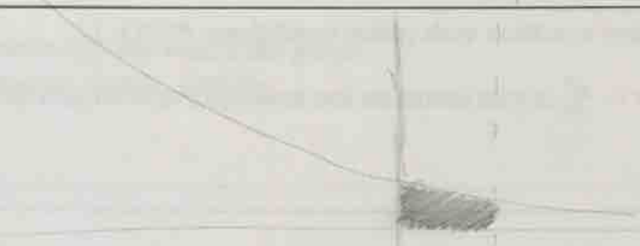
$= \frac{2\pi^2 - 32}{\sqrt{\pi}}$

3. Let $R$ be the region in the first quadrant bounded by the graph of $f(x) = \sqrt{1+e^{-x}}$, the line $x = 2$ and the $x$-axis.

   (a) Write an integral that gives the area of $R$.

   (b) Use the Trapezoidal Rule with $n = 4$ to approximate the area. You must show the numbers that lead to your answer.

   (c) Find the volume of the solid produced when $R$ is revolved about the $x$-axis.



$\frac{2-0}{2(4)} \cdot \frac{2}{8} = \frac{1}{4}$

$1+e^{-x}=0$

$e^{-x}=-1$

$\int_0^2 \sqrt{1+e^{-x}}\,dx$

$\frac{2}{8} = \frac{1}{4}$

$1+e^0 = \sqrt{2}$

$\frac{2-0}{2(4)}\left[f(0) + 2[f(1/2) + f(1) + f(1.5)] + f(2)\right]$

$= \frac{1}{4}\left(\sqrt{2} + 2[\sqrt{1+e^{-1/2}} + \sqrt{1+e^{-1}} + \sqrt{1+e^{-1.5}}] + \sqrt{1+e^{-2}}\right)$

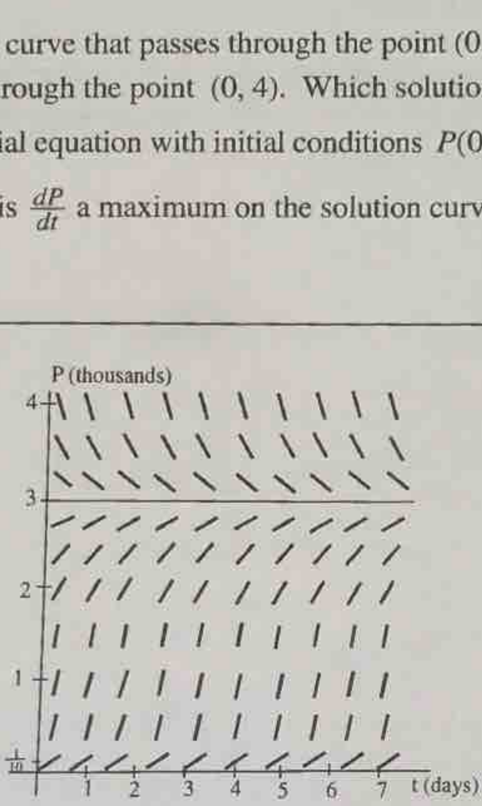
A CALCULATOR MAY **NOT** BE USED ON THIS PART OF THE EXAMINATION.
DURING THIS 45 MINUTES YOU MAY GO BACK AND WORK ON PART A.

4. Suppose that a population of bacteria grows according to the logistic equation $\frac{dP}{dt} = 2P(3-P)$, where $P$ is the population measured in thousands and $t$ is time measured in days. A slope field for this equation is given below.

   (a) Sketch the solution curve that passes through the point $(0, \frac{1}{10})$ and sketch the solution curve that passes through the point $(0, 4)$. Which solution has an inflection point?

   (b) Solve the differential equation with initial conditions $P(0) = 1$.

   (c) At what value of $t$ is $\frac{dP}{dt}$ a maximum on the solution curve in part (b)? Justify your answer.




$\frac{dP}{dr} = 2P(3-P)$

$\int \frac{dP}{2P(3-P)} = \int dt$

$\int \frac{dP}{6P-2P^2} = \int dt$

$= 2\int \frac{1}{2P} + \frac{1}{3-P} = \int d$

$\frac{\ln P}{2} - \ln(3-P) = t + c$

$P(0) = 1$

$\ln\left(\frac{P}{2(3-P)}\right) = t + c$

$\frac{P}{6-2P} = ce^{t}$

$P = ce^{t}(6-2P)$

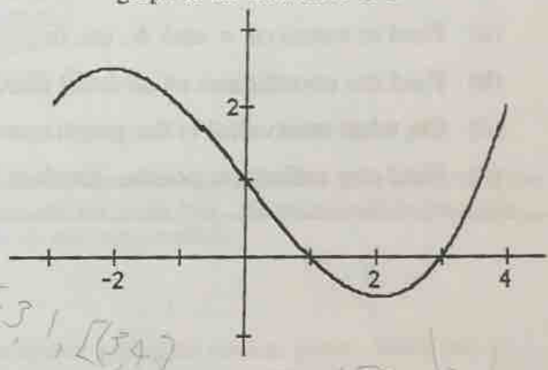
5. A function $F$ is defined for $x$ on the closed interval $[-3, 4]$. The graph of the *derivative* of $F$ is shown at the right.

   (a) Find the interval(s) for which the graph of $F$ is increasing. Explain briefly.

   (b) Find the possible $x$-coordinates for the absolute minimum value and the absolute maximum value of $F$ on $[-3, 4]$.

   (c) Find the interval(s) for which the graph of $F$ is concave down. Explain briefly.

graph of the derivative of $F$




Rel max at $x=1$
rel min at $x=3$

Min at $x=7$

6. Let $f(x) = ax + \dfrac{b}{x}$ where $a$ and $b$ are positive constants.

   (a) Find in terms of $a$ and $b$, the intervals on which $f$ is increasing.

   (b) Find the coordinates of all local maximum and minimum points.

   (c) On what interval(s) is the graph concave up?

   (d) Find any inflection points. Explain briefly.

$f(x) = ax + \dfrac{b}{x}$ $\qquad x > 0$

EXAM III
CALCULUS AB
SECTION I PART A
MULTIPLE CHOICE
NO CALCULATORS
Time–55 minutes
Number of questions–28

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

1. Which of the following is a function with a vertical tangent at $x = 0$?

(A) $f(x) = x^3$ (B) $f(x) = \sqrt[3]{x}$ (C) $f(x) = \frac{1}{x}$ (D) $f(x) = \sin x$ (E) $f(x) = \tan x$

$= x^{1/3}$
$= \frac{1}{3}x^{-2/3}$
$= \frac{1}{3x^{2/3}}$

$= x^{-1}$
$f'(x) = -x^{-2}$
$= -\frac{1}{x^2}$

$f'(x) = \cos x$

$f'(x) = \sec^2 x$
$= \frac{1}{\cos^2 x}$
$\cos x = 0$
$x = \pi/2$

Ans $\square$

2. $\displaystyle\int_0^5 \frac{dx}{\sqrt{1+3x}} =$

(A) 4 (B) $\frac{8}{3}$ (C) 2 (D) $\frac{16}{9}$ (E) 1

$= \left[\frac{(1+3x)^{1/2}}{3\cdot\frac{1}{2}}\right]_0^5$

$= \frac{\sqrt{16}}{\frac{3}{2}} - \frac{1}{\frac{3}{2}} = \frac{8}{3} - \frac{2}{3} = \frac{6}{3} = 2$

Ans $\square$

3. Which function is NOT continuous everywhere?

(A) $y = |x|$

(B) $y = x^{2/3}$ $\sqrt[3]{x^2}$

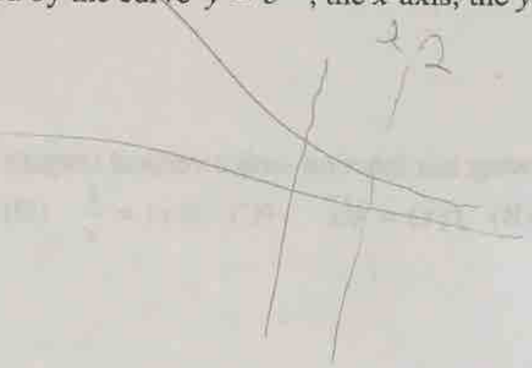
(C) $y = \sqrt{x^2 + 1}$

(D) $y = \dfrac{x}{x^2 + 1}$

(E) $y = \dfrac{4x}{(x+1)^2}$

Ans

4. The area of the region bounded by the curve $y = e^{-x}$, the $x$-axis, the $y$-axis and the line $x = 2$ is equal to



(A) 1 units$^2$

(B) 2 units$^2$

(C) $\ln e^x$ units$^2$

(D) $\dfrac{1}{e^2} - 1$ units$^2$

(E) $1 - \dfrac{1}{e^2}$ units$^2$

Ans

5. If $g(x) = x + \cos x$, then $\displaystyle\lim_{h \to 0} \frac{g(x+h) - g(x)}{h} =$

(A) $\sin x + \cos x$ $g'(x) = 1 - \sin x$

(B) $\sin x - \cos x$

(C) $1 - \sin x$

(D) $1 - \cos x$

(E) $x^2 - \sin x$

Ans

6. $\int_0^4 \frac{2x}{x^2+9}\,dx =$

(A) 25

(B) 16

(C) $\ln \frac{25}{9}$

(D) $\ln 4$

(E) $\ln \frac{5}{3}$

$\ln(x^2+9)$
$\ln(25) - \ln 9$
$= \ln \frac{25}{9}$

Ans

7. A function $g$ is defined for all real numbers and has the following property: $g(a+b) - g(a) = 4ab + 2b^2$. Find $g'(x)$.

(A) 4

(B) $-4$

(C) $2x^2$

(D) $4x$

(E) does not exist

$g(x+b) - g(x) = 4xb + 2b^2 = b(4x+2b)$
$\frac{g(x+b) - g(x)}{b} = 4x + 2b = 4x + 2b$
$\lim_{b \to 0} \left[ \frac{g(x+b) - g(x)}{b} \right] = 4x$

Ans

8. Given the function defined by $f(x) = x^5 - 5x^4 + 3$, find all values of $x$ for which the graph of $f$ is concave up.

(A) $x > 0$

(B) $x > 3$

(C) $0 < x < 3$

(D) $x < 0$ or $x > 3$

(E) $x < 0$ or $x > 5$

$y = x^5 - 5x^4 + 3$
$y' = 5x^4 - 20x^3$

Ans

9. If $f(x) = \ln\left(e^{x^2}\right)$, find the average value of the function $f$ on the interval $0 \le x \le 3$.

(A) $\frac{1}{3}$ (B) 1 (C) 3 (D) 9 (E) 27

$\frac{1}{3}\int_0^3 \ln(e^{x^2})\,dx$

$u = \ln(e^{x^2})$, $\frac{du}{dx} = \frac{2xe^{x^2}}{(e^{x^2})}$, $\frac{dv}{dx} = 1$, $v = x$

$= \ln(e^{x^2})x - \int x(2x)\,dx$

$= x\ln(e^{x^2}) - \int 2x^2$

$= \frac{1}{3}\left[x\ln e^{x^2} - \frac{2}{3}x^3\right]_0^3$

$= \frac{1}{3}\left[3\ln e^9 - \frac{2}{3}(27)\right]$

$= \frac{1}{3}(27-18) = \frac{1}{3}(9) = 3$

Ans

10. A particle starts at (5, 0) when $t = 0$ and moves along the $x$-axis in such a way that at time $t > 0$ its velocity is given by $v(t) = \dfrac{1}{1+t}$. Determine the position of the particle at $t = 3$.

(A) $\frac{97}{16}$

(B) $\frac{95}{16}$

(C) $\frac{79}{16}$

(D) $1 + \ln 4$

(E) $5 + \ln 4$

$s(t) = 5 + \int_0^3 \frac{1}{1+t}\,dt$

$= 5 + [\ln|1+t|]_0^3$

$= 5 + \ln 4 - \ln 1$

$= 5 + \ln 4$

Ans

11. If $g(x) = \sqrt[3]{x-1}$ and $f$ is the inverse function of $g$, then $f'(x) =$

(A) $3x^2$

(B) $3(x-1)^2$

(C) $-\frac{1}{3}(x-1)^{-4/3}$

(D) $\frac{1}{3}(x-1)^{2/3}$

(E) does not exist

$g(x) = (x-1)^{1/3}$

$x = (y-1)^{1/3}$

$x^3 = y-1$

$y = x^3+1$

$y' = 3x^2$

Ans

12. Suppose $F(x) = \int_0^{\cos x} \sqrt{1+t^3}\, dt$ for all real $x$, then $F'\left(\frac{\pi}{2}\right) =$

(A) $-1$
(B) $0$
(C) $\frac{1}{2}$
(D) $1$
(E) $\frac{\sqrt{3}}{2}$

$F'(x) = \sqrt{1+(\cos x)^3} \cdot -\sin x$
$F'\left(\frac{\pi}{2}\right) = \sqrt{1+0} \cdot -\sin\frac{\pi}{2}$
$= 1 \cdot (-1)$
$= -1$

Ans

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13. If the line $3x - y + 2 = 0$ is tangent in the first quadrant to the curve $y = x^3 + k$, then $k =$

(A) 5
(B) $-5$
(C) 4
(D) 1
(E) $-1$

$y = 3x + 2$ $\quad y' = 3x^2$
$3x^2 = 3$
$x^2 = 1$
$x = \pm 1$
$x = 1$
$3(1) - y + 2 = 0$
$y = 5$
$5 = 1 + k$
$k = 4$

Ans

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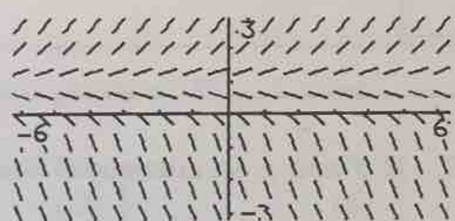
14. The slope field for a differential equation $\frac{dy}{dx} = f(x, y)$ is given in the figure. Which of the following statements are true?




I. A solution curve that contains the point (0, 2) also contains the point (–2, 0).

II. As $y$ approaches 1, the rate of change of $y$ approaches zero.

III. All solution curves for the differential equation have the same slope for a given value of $y$

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, III

Ans

15. $\frac{d}{dx}[\text{Arctan } 3x] =$

(A) $\frac{1}{1+9x^2}$

(B) $\frac{3}{1+9x^2}$

(C) $\frac{3}{\sqrt{4x^2-1}}$

(D) $\frac{3}{1+3x}$

(E) none of the above

$\frac{d}{dx} \tan^{-1} 3x = \frac{3}{1+(3x)^2}$

$= \frac{3}{1+9x^2}$

Ans

16. $\lim_{x \to 1} \frac{x^2+2x-3}{x^2-1} =$

(A) –2

(B) –1

(C) 10

(D) 1

(E) 2

$\frac{\cancel{(x-1)}(x+3)}{(x+1)\cancel{(x-1)}} \lim_{x \to 1} \frac{x+3}{x+1} = \frac{4}{2} = 2$

Ans

17. If $f$ and $g$ are continuous functions such that $g'(x) = f(x)$ for all $x$, then $\int_2^3 f(x)\,dx =$

(A) $g'(2) - g'(3)$

(B) $g'(3) - g'(2)$

(C) $g(3) - g(2)$

(D) $f(3) - f(2)$

(E) $f'(3) - f'(2)$

$G(3) - G(2)$

$g(3) - g(2)$

Ans

18. Let $y = 2e^{\cos x}$. Both $x$ and $y$ vary with time in such a way that $y$ increases at the constant rate of 5 units per second. The rate at which $x$ is changing when $x = \frac{\pi}{2}$ is

(A) 10 units/sec

(B) −10 units/sec

(C) −2.5 units/sec

(D) 2.5 units/sec

(E) −0.4 units/sec

$y = 2e^{\cos x}$ $\frac{dy}{dt} = 5$ $\frac{dx}{dt}$

$\frac{dy}{dt} = -2\sin x e^{\cos x}\frac{dx}{dt}$

$5 = -2\sin\frac{\pi}{2}e^{0}\frac{dx}{dt}$

$5 = -2(1)\frac{dx}{dt}$

$\frac{dx}{dt} = -\frac{5}{2} = -2.5$

Ans

19. $\int_1^2 \frac{dx}{x^3} =$

(A) $\frac{3}{8}$

(B) $-\frac{3}{8}$

(C) $\frac{15}{64}$

(D) $\frac{3}{4}$

(E) $\frac{15}{16}$

$\int^2 x^{-3}dx$

$= \left[\frac{x^{-2}}{-2}\right]_1^2 = \left[-\frac{1}{2x^2}\right]_1^2$

$= -\frac{1}{8} + \frac{1}{2}$

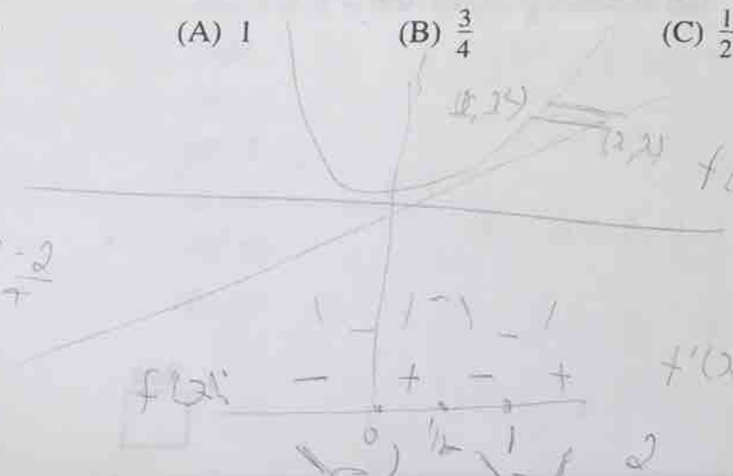
$= -\frac{1}{8} + \frac{4}{8}$

$= \frac{3}{8}$

Ans

20. The maximum distance, measured horizontally, between the graphs of $f(x) = x$ and $g(x) = x^2$ for $0 \le x \le 1$, is

(A) 1 (B) $\frac{3}{4}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$ (E) $\frac{1}{8}$



$(x, x)$, $(x, x^2)$ $(x^2-x)(x^2-x)$

$f(x) = \sqrt{(x^2-x)^2 + (x-x)^2}$ $x = \frac{1}{2}$

$= \sqrt{x^4 - 2x^3 + x^2}$

$f'(x) = 4x^3 - 6x^2 + 2x = 0$

$2x(2x^2 - 3x + 1) = 0$

$x = 0,\ x = 1,\ x = \frac{1}{2}$

$f(x) = \sqrt{(\frac{1}{4} - \frac{1}{2})^2 + 0^2} = \sqrt{\frac{1}{16}} = \frac{1}{4}$

Ans

21. Let $f$ be the function defined by $f(x) = \begin{cases} x+1 & \text{for } x < 0 \\ 1+\sin \pi x & \text{for } x \geq 0. \end{cases}$ Then $\int_{-1}^{1} f(x)\,dx =$

(A) $\frac{3}{2}$

(B) $\frac{3}{2} - \frac{2}{\pi}$

(C) $\frac{1}{2} - \frac{2}{\pi}$

(D) $\frac{3}{2} + \frac{2}{\pi}$

(E) $\frac{1}{2} + \frac{2}{\pi}$

$\int_0^1 1+\sin \pi x + \int_{-1}^0 x+1$

$= \left[x - \frac{\cos \pi x}{\pi}\right]_0^1 + \left[\frac{x^2}{2} + x\right]_{-1}^0$

$= 1 - \frac{\cos \pi}{\pi} + \frac{\cos 0}{\pi} + \left[0 - \left(\frac{1}{2} - 1\right)\right]$

$= 1 + \frac{1}{\pi} + \frac{1}{\pi} + \frac{1}{2}$

$= \frac{2}{\pi} + \frac{3}{2}$

Ans $\square$

22. If $x^2 + 2xy - 3y = 3$, then the value of $\frac{dy}{dx}$ at $x = 2$ is

(A) 1

(B) 2

(C) $-2$

(D) $\frac{10}{3}$

(E) $-\frac{1}{2}$

Ans $\square$

23. Let $f$ be the function defined by $f(x) = x^{2/3}(5-2x)$. $f$ is increasing on the interval

(A) $x < -\frac{5}{2}$ (B) $x > 0$ (C) $x < 1$ (D) $0 < x < \frac{5}{8}$ (E) $0 < x < 1$

$f'(x) = \frac{2}{3}x^{-1/3}(5-2x) + x^{2/3}(-2)$

$= \frac{2}{3x^{1/3}}(5-2x) - 2x^{2/3}$

$= 2x^{-1/3}\left(\frac{1}{3}(5-2x) - x\right)$

$= \frac{2}{x^{1/3}}\left[\frac{5}{3} - \frac{5}{3}x\right) = 0$

$\frac{5}{3}x = \frac{5}{3}$

$x = 1$

$f'(x)$: $-$ $+$ $-$ ; 0, 1

Ans $\square$

24. Let $R$ be the region in the first quadrant bounded by the $x$-axis and the curve $y = 2x - x^2$. The volume produced when $R$ is revolved about the $x$-axis is

(A) $\frac{16\pi}{15}$ units$^3$ (B) $\frac{8\pi}{3}$ units$^3$ (C) $\frac{4\pi}{3}$ units$^3$ (D) $16\pi$ units$^3$ (E) $8\pi$ units$^3$

$y = x(2-x)$

$V = \pi\int_0^2 (2x-x^2)^2$

$= \pi\int_0^2 (4x^2 - 4x^3 + x^4)\,dx$

$= \pi\left[\frac{4x^3}{3} - x^4 + \frac{x^5}{5}\right]_0^2$

$R(x) = 2x - x^2$

$(2x-x^2)(2x-x^2)$

$4x^2 - 4x^3 + x^4$

Ans

25. What are all values of $k$ for which the graph of $y = 2x^3 + 3x^2 + k$ will have three distinct $x$-intercepts?

$= \frac{32}{3} - 16 + \frac{32}{5}$

$= \pi\left(\frac{160-240+96}{15}\right)$

$= \pi\left(\frac{16}{15}\right)$

(A) all $k < 0$

(B) all $k > -1$

(C) all $k$

(D) $-1 < k < 0$

(E) $0 < k < 1$

$2x^3 + 3x^2 + k = 0$

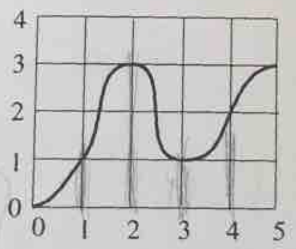
$(x-)(x-)(x$

$2x^3 + 3x^2 + k = 0$

Ans

26. Use the Trapezoid Rule with $n = 4$ to approximate the integral $\int_1^5 f(x)\,dx$ for the function $f$ whose graph is shown at the right.

graph of $y = f(x)$



(A) 7

(B) 8

(C) 9

(D) 10

(E) 11

$\frac{5-1}{8}\left[f(1)\ 2[f(2)+f(3)+f(4)] + f(5)\right]$

$= \frac{1}{2}\left[1 + 2(3+1+2) + 3\right]$

$= \frac{1}{2}\left[1+3+12\right]$

$= \frac{1}{2}(16)$

$= 8$

Ans

27. A point moves so that $x$, its distance from the origin at time $t$, $t \geq 0$ is given by: $x(t) = \cos^3 t$. The first time interval in which the point is moving to the right is

(A) $0 < t < \frac{\pi}{2}$

(B) $\frac{\pi}{2} < t < \pi$

(C) $\pi < t < \frac{3\pi}{2}$

(D) $\frac{3\pi}{2} < t < 2\pi$

(E) none of these

$v(t) = 3\cos^2 t(-\sin t)$  $v(t)$:

$-\quad -\quad +$

$0 \quad \frac{\pi}{2} \quad \pi \quad \frac{3\pi}{2} \quad 2\pi$

$-3\sin t\cos^2 t = 0$

$\sin t\cos^2 t = 0$

$\sin t = 0$  $\cos^2 t = 0$

$\sin t = 0$  $\cos t = 0$

$t = 0, \pi, 2\pi$  $t = \frac{\pi}{2}, \frac{3\pi}{2}$

Ans

28. The function $f$, for which $f'(x) = (x-2)^2(x-7)^3$, has an inflection point where $x =$

(A) 4 only

(B) 7 only

(C) 2 and 4 only

(D) 2 and 7 only

(E) 2 and 4 and 7

Ans

EXAM III
CALCULUS AB
SECTION I PART B
MULTIPLE-CHOICE
CALCULATORS
Time–50 minutes
Number of questions–17

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS IN THIS PART OF THE EXAMINATION.

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
(2) Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

1. The derivative of the function $g$ is $g'(x) = \cos(\sin x)$. At the point where $x = 0$ the graph of $g$

I. is increasing, II. is concave down, III. attains a relative maximum point.

(A) I only (B) II only (C) III only (D) I and III only (E) I, II, III

$g'(x) = \cos(\sin x)$
$g'(0) = \cos(0) = 1$
$g''(x) = -\cos x \sin(\sin x)$
$g''(0) = -\cos 0 \cdot \sin 0 = 0$

$\cos(\sin x) = 0$
$\sin x = \frac{\pi}{2}, \frac{3\pi}{2}$
$x =$

Ans

2. The approximate *average* rate of change of the function $f(x) = \int_0^x \sin(t^2)\, dt$ over the interval [1, 3] is

(A) 0.19 (B) 0.23 (C) 0.27 (D) 0.31 (E) 0.35

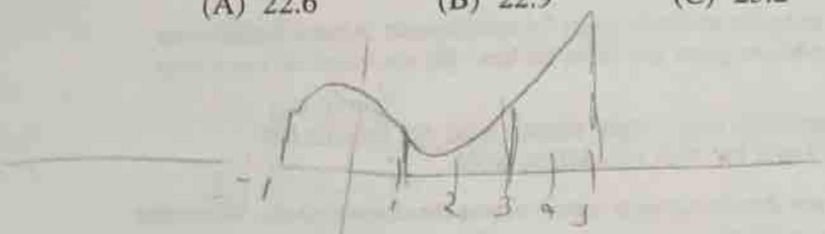
$\frac{F(3) - F(1)}{3-1}$
$= \frac{0.463}{2} =$

$f(3) = \int_0^3 \sin t^2 =$
$f(1) = \int_0^1 \sin(t^2)$

Ans

3. When $\int_{-1}^{5} \sqrt{x^3 - x + 1}\, dx$ is approximated by using the mid-points of 3 rectangles of equal width, then the approximation is nearest to

(A) 22.6 (B) 22.9 (C) 23.2 (D) 23.5 (E) 23.8




$M = \frac{5+1}{3}\left(f(0)+f(2)+f(4)\right)$

$= 2\left[1+\sqrt{7}+\sqrt{61}\right] = 22.9$

Ans

4. Find the total area between the graph of the curve $y = x^3 - 5x^2 + 4x$ and the $x$-axis.

(A) 11.74 units$^2$

(B) 11.77 units$^2$

(C) 11.80 units$^2$

(D) 11.83 units$^2$

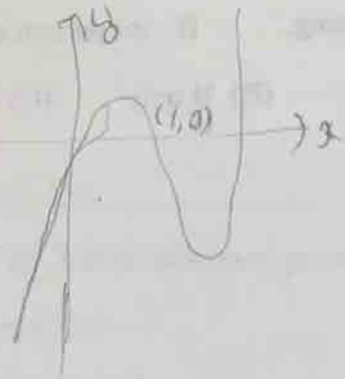
(E) 11.86 units$^2$

$x(x^2-5x+4)$

$x(x-1)(x-4)=0$

$x=0, 1, 4$

$\int_0^1 4dx - \int_1^4 4dx$




Ans

5. The graph of $y = \frac{\sin x}{x}$ has

I. a vertical asymptote at $x = 0$

II. a horizontal asymptote at $y = 0$

III. an infinite number of zeros

(A) I only

(B) II only

(C) III only

(D) I and III only

(E) II and III only

$\lim_{x\to\infty} \frac{\sin\infty}{\infty} = \frac{\text{undefined}}{\infty} = 0$

$-1 \le \sin x \le 1$

$\frac{-1}{x} \le \frac{\sin x}{x} \le \frac{1}{x}$

$\lim_{x\to\infty} \frac{-1}{x} = \frac{-1}{\infty} = 0$

$\frac{\sin x}{x} = 0$

$\sin x = 0$

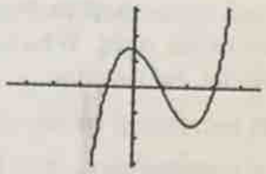
$x = 0, \pi, 2\pi$

Ans

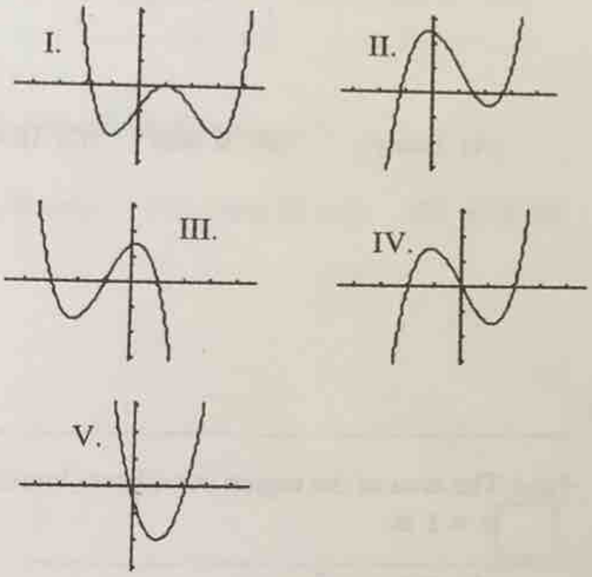
6. The graph of the function $f$ is shown at the right. The graphs of the five functions:

$y = f(x+1)$, IV
$y = f(x)+1$, II
$y = f(-x)$, III
$y = f'(x)$ and V
$y = \int_0^x f(t)\,dt$ I

are shown in the *wrong* order.



the graph of $f$




The correct order is

(A) II, IV, III, V, I

(B) IV, II, III, I, V

(C) IV, II, III, V, I

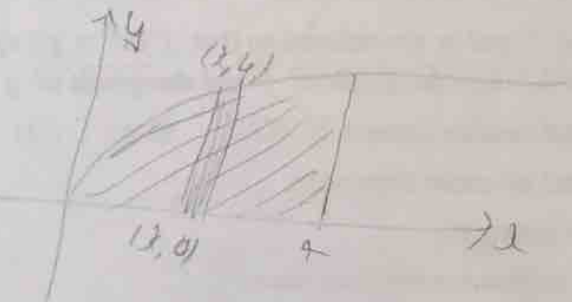
(D) IV, III, II, V, I

(E) II, IV, III, I, V

Ans

7. The region in the first quadrant bounded above by the graph of $y = \sqrt{x}$ and below by the interval $[0, 4]$ is revolved about the $x$-axis. If a plane perpendicular to the $x$-axis at the point where $x = k$ divides the solid into parts of equal volume, then $k =$

(A) 2.77 (B) 2.80 (C) 2.83 (D) 2.86 (E) 2.89




$r(x) = y - 0 = \sqrt{x}$

$\pi \int_0^k x\,dx = \pi \int_k^4 x\,dx$

$\left[\frac{x^2}{2}\right]_0^k = \left[\frac{x^2}{2}\right]_k^4$

Ans

$\frac{k^2}{2} = \frac{16}{2} - \frac{k^2}{2}$ $k^2 = \frac{16}{2}$ $k = \frac{4}{\sqrt{2}}$

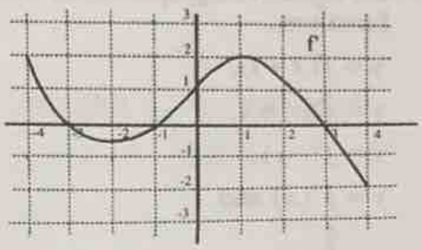
8. The graph of the **derivative** of a function $f$ is shown to the right. Which of the following is true about the function $f$?

   I. $f$ is increasing on the interval $(-2, 1)$. No

   II. $f$ is continuous at $x = 0$. Yes

   III. $f$ has an inflection point at $x = -2$. Yes

graph of the derivative of $f$




(A) I only (B) II only (C) III only (D) II and III only (E) I, II, III

Ans

9. The area of the region completely bounded by the curve $y = -x^2 + 2x + 4$ and the line $y = 1$ is

$y = -x^2+2x+4$
$= -(x^2-2x-4)$

(A) 8.7 units$^2$

(B) 9.7 units$^2$

(C) 10.7 units$^2$

(D) 11.7 units$^2$

(E) 12.7 units$^2$

$\int_{-1}^{3} (y-1)\,dx$ =

Ans

737

10. If functions $f$ and $g$ are defined so that $f'(x) = g'(x)$ for all real numbers $x$ with $f(1) = 2$ and $g(1) = 3$, then the graph of $f$ and the graph of $g$

(A) intersect exactly once;

(B) intersect no more than once;

(C) do not intersect;

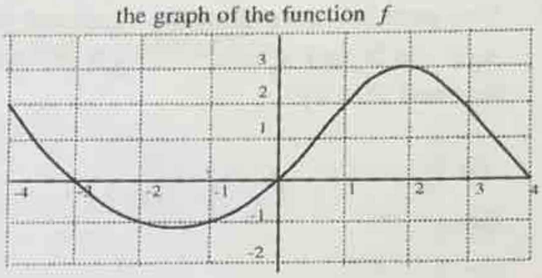
(D) could intersect more than once;

(E) have a common tangent at each point of tangency.

Ans

11. The graph of a function $f$ whose domain is the interval $[-4, 4]$ is shown in the figure. Which of the following statements are true?

the graph of the function $f$




I. The average rate of change of $f$ over the interval from $x = -2$ to $x = 3$ is $\frac{1}{5}$. No

II. The slope of the tangent line at the point where $x = 2$ is 0. Yes

III. The left-sum approximation of $\int_{-1}^{3} f(t)\,dt$ with 4 equal subdivisions is 4. Yes

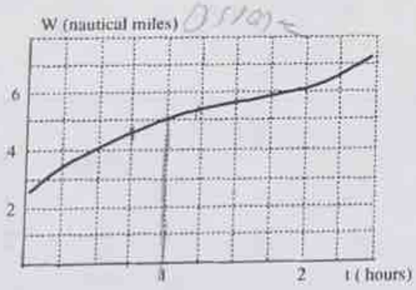
(A) I only (B) I and II only (C) II and III only (D) I and III only (E) I, II, III

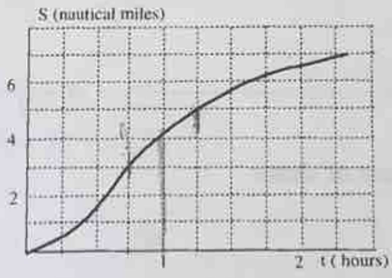
$\frac{f(3)-f(-2)}{5} = \frac{2-(-1)}{5} = \frac{3}{5}$

$\frac{3+1}{4}\left[f(-1)+f(0)+f(1)+f(2)\right]$
$= -1+0+2+3$
$= 4$

Ans

12. One ship traveling west is $W(t)$ nautical miles west of a lighthouse and a second ship traveling south is $S(t)$ nautical miles south of the lighthouse at time $t$ (hours). The graphs of $W$ and $S$ are shown below. At what approximate rate is the distance between the ships increasing at $t = 1$? (nautical miles per hour = knots)



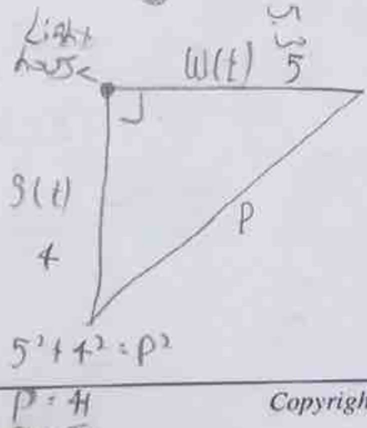




$\frac{dw}{dt} = \frac{5.5-4.5}{1.25-0.75} = 2$

$\frac{ds}{dt} = \frac{5-3}{1.25-0.75} = 4$

Want $\frac{dp}{dt}$

(A) 1 knot (B) 4 knots (C) 7 knots (D) 10 knots (E) 13 knots




$5^2 + 4^2 = p^2$
$p = 41$
$p = \sqrt{41}$

$p^2 = w^2 + s^2$

$2p\frac{dp}{dt} = 2w\frac{dw}{dt} + 2s\frac{ds}{dt}$

$2\sqrt{41}\frac{dp}{dt} = 2(5)(2) + 2(4)(4)$

$\frac{dp}{dt} = \frac{26}{\sqrt{41}} = 4.061$

Ans

13. Two particles move along the $x$-axis and their positions at time $0 \le t \le 2\pi$ are given by $x_1 = \cos 2t$ and $x_2 = e^{(t-3)/2} - 0.75$. For how many values of $t$ do the two particles have the same velocity?

$\frac{t-3}{2}$ $\quad \frac{t}{2} - \frac{3}{2}$

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

$V_1 = -2\sin 2t \qquad V_2 = \frac{1}{2} e^{(t-3)/2}$

$-2\sin 2t = \frac{1}{2} e^{(t-3)/2}$

$-4\sin 2t = e^{(t-3)/2}$

...

$f'(2) =$

Ans $\square$

14. The line $x - 2y + 9 = 0$ is tangent to the graph of $y = f(x)$ at $(3, 6)$ and is also parallel to the line through $(1, f(1))$ and $(5, f(5))$. If $f$ is differentiable on the closed interval $[1, 5]$ and $f(1) = 2$, find $f(5)$.

(A) 2

(B) 3

(C) 4

(D) 5

(E) none of these

Slope line $= \frac{f(5) - f(1)}{5-1} = f'(2)$

$2y = x + 9$

$y = \frac{1}{2}x + \frac{9}{2}$

$\frac{f(5) - f(1)}{4} = \frac{1}{2}$

$\frac{f(5) - 2}{4} = \frac{1}{2}$

$f(5) - 2 = 2$

$f(5) = 4$

$\frac{f(5) - f(1)}{5-1} = f'(2)$

$\frac{f(5) - f(1)}{4} =$

$2x \cdot f(3)$

$f'(x^2) \cdot 2x$

$\frac{d}{dx^2} f(x) = f(3x)$

Ans $\square$

15. If $\frac{d}{dx}[f(x)] = g(x)$ and $\frac{d}{dx}[g(x)] = f(3x)$, then $\frac{d^2}{dx^2}\left[f(x^2)\right]$ is

(A) $4x^2 f(3x^2) + 2g(x^2)$

(B) $f(3x^2)$

(C) $f(x^4)$

(D) $2xf(3x^2) + 2g(x^2)$

(E) $2xf(3x^2)$

$\frac{d}{dx}[f(x)] = g(x)$

$f'(x) = g(x)$

$\frac{d}{dx}[g(x)] = f(3x)$

$g'(x) = f(3x)$

$f''(x) = g'(x) = f(3x)$

$\frac{d^2}{dx^2}[f(x^2)] =$

$\frac{d}{dx}[f'(x^2)\cdot 2x]$

$f''(x^2)\cdot 4x^2 + 2f'(x^2)$

$f(3x^2)\cdot 4x^2 + 2g(x^2)$

Ans $\square$

16. The point $(1, 4)$ lies on the graph of an equation $y = f(x)$ for which $\frac{dy}{dx} = 3\sqrt{xy}$ where $x \geq 0$ and $y \geq 0$. When $x = 0$ the value of $y$ is

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

$\frac{dy}{dx} = 3\sqrt{xy}$

$dy = 3\sqrt{xy}\,dx$

$\frac{dy}{\sqrt{y}} = 3\sqrt{x}\,dx$

$2y^{1/2} = 2x^{3/2} + c$

$2(2) = 2(1) + c$

$c = 2$

$y^{-1/2}$

$\frac{x^{3/2}}{3/2}$

$2y^{1/2} = 2x^{3/2} + 2$

$x = 0$

$2\sqrt{y} = 2$

$\sqrt{y} = 1$

$y = 1$

Ans $\square$

17. Find $\frac{dy}{dx}$ for $e^y = xy$.

(A) $\ln x + \ln y$

(B) $\frac{x+y}{xy}$

(C) $\frac{xy}{x+y}$

(D) $\frac{xy - x}{y}$

(E) $\frac{y}{xy - x}$

$e^y = xy$

$y'e^y = xy' + y$

$y'(e^y - x) = y$

$y' = \frac{y}{e^y - x}$

$= \frac{y}{xy - x}$

Ans $\square$

EXAM III
CALCULUS AB
SECTION II, PART A
Time–45 minutes
Number of questions–3

A GRAPHING CALCULATOR IS REQUIRED ON THIS PART OF THE EXAMINATION.

- Before you begin Part A of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems and you will be able to come back to Part A (without a calculator), if you have time after Part B. All problems are given equal weight, but the parts of a particular solution are not necessarily given equal weight.
- You should write all work for each problem in the space provided. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed out work will not be graded.
- SHOW ALL YOUR WORK. You will be graded on the correctness and completeness of your methods as well as the accuracy of your final answers. Correct answers without supporting work may not receive full credit.
- Justifications require that you give mathematical (noncalculator) reasons and that you clearly identify functions, graphs, tables, or other objects you use.
- You are permitted to use your calculator in Part A to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate in your exam booklet the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.
- Your work must be expressed in mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt($X^2$, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.
- Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

THE EXAM BEGINS ON THE NEXT PAGE

PLEASE TURN OVER

1. Let $R$ be the region in the first quadrant bounded above by the graph of $f(x) = 3\cos x$ and below by the graph of $g(x) = e^{x^2}$.

   (a) Find the area of region $R$.

   (b) Set up, but do not integrate an integral expression in terms of a single variable for the volume of the solid generated when $R$ is revolved about the x-axis.

   (c) Let the base of a solid be the region $R$. If all cross sections perpendicular to the $x$-axis are squares, set up, but do not integrate an integral expression in terms of a single variable for the volume of the solid.

2. Let $f$ be the function defined by $f(x) = \ln(x+1) - \sin^2 x$ for $0 \le x \le 3$.

(a) Find the $x$-intercepts of the graph of $f$.

(b) Find the intervals on which $f$ is increasing.

(c) Find the absolute maximum and the absolute minimum value of $f$. Justify your answer.

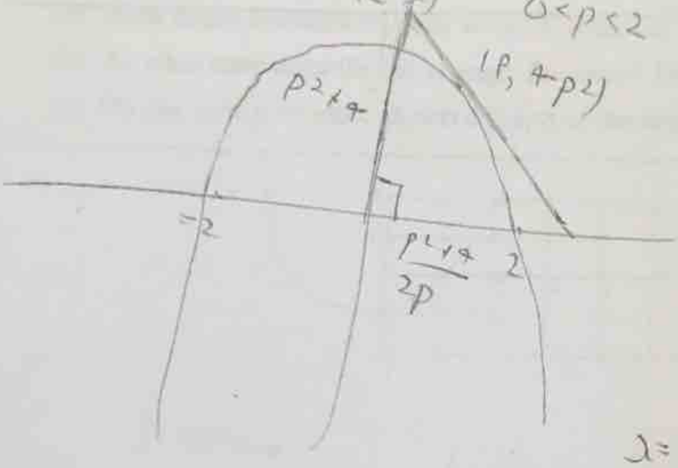
$\ln(x+1) = -\sin^2 x$

3. Let $f(x) = 4 - x^2$. For $0 < p < 2$, let $A(p)$ be the area of the triangle formed by the coordinate axes and the line tangent to the graph of $f$ at the point $(p, 4 - p^2)$.

(a) Find $A(2)$.

(b) For what value of $p$ is $A(p)$ a minimum?

$f(x) = (2+x)(2-x)$ $\quad 0<p<2$




Equation of tangent:

$f'(x) = -2x$

$y-(4-p^2) = -2p(x-p)$

$y-4+p^2 = -2px+2p^2$

$y = p^2-2px+4$

$x=0$: $y = p^2+4$

$y=0$: $p^2-2px+4=0$

$2px = p^2+4$

$x = \frac{p^2+4}{2p}$

$Area = \frac{1}{2}\left(\frac{p^2+4}{2p}\right)(p^2+4)$

$A(2) = \frac{1}{2}\left(\frac{8}{4}\right)(8)$

$= \frac{64}{8} = 8$

$A = \frac{1}{4p}(p^2+4)^2$

$A' = \frac{1}{4p}\left[2(p^2+4)2p\right] + \frac{1}{4}(p^2+4)^2 = 0$

$p^2+4 + \frac{1}{4}(p^2+4)^2 = 0$

$p^2+4\left[1+\frac{1}{4}(p^2+4)\right] = 0$

$(p^2+4)\left(\frac{1}{4}p^2+2\right] = 0$

$p = \pm 2 \quad \frac{1}{4}p^2 = -2$

$p^2 = -\frac{3}{2}$

A CALCULATOR MAY **NOT** BE USED ON THIS PART OF THE EXAMINATION.
DURING THIS 45 MINUTES YOU MAY GO BACK AND WORK ON PART A.

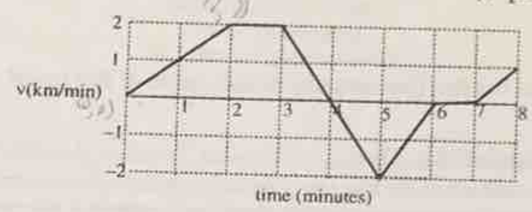
4. Let $f(x) = x^3 + px^2 + qx$.

   (a) Find the values of $p$ and $q$ so that $f(-1) = -8$ and $f'(-1) = 12$.

   (b) Find the value of $p$ so that the graph of $f$ changes concavity at $x = 2$.

   (c) Under what conditions on $p$ and $q$ will the graph of $f$ be increasing everywhere.

5. A car is moving along a straight road from $A$ to $B$, starting from $A$ at time $t = 0$. Below is a graph of the car's velocity (positive direction from $A$ to $B$), plotted against time.




(a) How many kilometers away from $A$ is the car at time $t = 6$? 3 km

(b) At what time does the car change direction? Explain briefly.

(c) On the axes provided, sketch a graph of the acceleration of the car.

Distance $= \int_0^6 v(t)$

$= \frac{1}{2}(2)(2) + \frac{1}{2}(1)(2) + 1(2) + \frac{1}{2}(1+2)(1) + \frac{1}{2}(1)(1)$

$= 2 + 1 + 2 + \frac{3}{2} + \frac{1}{2}$

$= 1 + 2$

$= 3$

6. Consider the curve given by the equation $y^3 - 3xy = 2$.

(a) Find $\dfrac{dy}{dx}$.

(b) Write an equation for the line tangent to the curve at the point $(1, 2)$.

(c) Find $\dfrac{d^2y}{dx^2}$ at the point $(1, 2)$.

EXAM IV
CALCULUS AB
SECTION I PART A
MULTIPLE CHOICE
NO CALCULATORS
Time–55 minutes
Number of questions–28

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

1. What is $\lim_{x\to 0}\left(\dfrac{\frac{1}{x-1}+1}{x}\right)$?

$\lim_{x\to 0} \dfrac{\frac{1}{x-1}+\frac{x-1}{x-1}}{x} = \dfrac{\frac{x}{x-1}}{x} = \dfrac{x}{x-1}\cdot\dfrac{1}{x} = \dfrac{1}{x-1} = \dfrac{1}{-1}$

(A) $-1$ (B) $0$ (C) $1$ (D) $2$ (E) the limit does not exist

Ans

2. $\displaystyle\int \frac{e^{\sqrt{x}}}{2\sqrt{x}}\,dx$

(A) $\ln\sqrt{x}+C$ (B) $x+C$ (C) $e^{x}+C$ (D) $\frac{1}{2}e^{2\sqrt{x}}+C$ (E) $e^{\sqrt{x}}+C$

$\dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$

$\int \dfrac{e^{\sqrt{x}}}{2}\cdot x^{-1/2}\,dx$

$= e^{\sqrt{x}} + C$

Ans

3. If $y = \dfrac{3}{4+x^2}$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{3}{2x}$

(B) $\dfrac{3x}{\left(1+x^2\right)^2}$

(C) $\dfrac{6x}{\left(4+x^2\right)^2}$

(D) $\dfrac{-6x}{\left(4+x^2\right)^2}$

(E) $\dfrac{-3}{\left(4+x^2\right)^2}$

$3(4+x^2)^{-1}$

$-3(4+x^2)^{-2}(2x)$

$= -\dfrac{6x}{(4+x^2)^2}$

Ans

4. If $F(x) = \displaystyle\int_1^x (\cos 6t + 1)\, dt$, then $F'(x) =$

(A) $\sin 6x + x$ (B) $\cos 6x + 1$ (C) $\frac{1}{6}\sin 6x + x$

(D) $-\frac{1}{6}\sin 6x + 1$ (E) $\sin 6x + 1$

Ans

5. Consider the curve $x + xy + 2y^2 = 6$. The slope of the line tangent to the curve at the point $(2,1)$ is

(A) $\frac{2}{3}$

(B) $\frac{1}{3}$

(C) $-\frac{1}{3}$

(D) $-\frac{1}{5}$

(E) $-\frac{3}{4}$

$1 + y + x\dfrac{dy}{dx} + 4y\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx}(x+4y) = -y-1$

$\dfrac{dy}{dx} = \dfrac{-y-1}{x+4y} = \dfrac{-1-1}{2+4} = \dfrac{-2}{6} = \dfrac{-1}{3}$

Ans

6. $\lim_{h\to 0} \dfrac{3\left(\frac{1}{2}+h\right)^5 - 3\left(\frac{1}{2}\right)^5}{h} =$ $f'$

$f(x) = 3(x^5)$
$f'(x) = 15x^4$
$f'(\frac{1}{2}) = 15(\frac{1}{2})^4$
$= 15(\frac{1}{16}) = \frac{15}{16}$

(A) 0

(B) 1

(C) $\frac{15}{16}$

(D) the limit does not exist

(E) the limit can not be determined

Ans

7. If $p(x) = (x-1)(x+k)$ and if the line tangent to the graph of $p$ at the point $(4, p(4))$ is parallel to the line $5x - y + 6 = 0$, then $k =$

(A) 2 $p(x) = (x-1)(x+k)$

(B) 1 $p(x) = x^2 + kx - x - k$

(C) 0 $p'(x) = 2x + k - 1$

(D) −1 $p'(4) = 8 + k - 1 = 7 + k$

(E) −2 $7 + k = 5$

$k = -2$

$5x - y + 6 = 0$
$y = 5x + 6$

Ans

8. If $\cos x = e^y$ and $0 < x < \frac{\pi}{2}$, what is $\frac{dy}{dx}$ in terms of $x$ ?

(A) $-\tan x$ $\cos x = e^y$

(B) $-\cot x$ $-\sin x = y' e^y$

(C) $\cot x$ $y' = \dfrac{-\sin x}{e^y}$

(D) $\tan x$

(E) $\csc x$ $= \dfrac{-\sin x}{e^{\ln \cos x}} = \dfrac{-\sin x}{\cos x} = -\tan x$

$\ln e^y = \ln \cos x$
$y = \ln \cos x$

Ans

9. At $t = 0$, a particle starts at the origin with a velocity of 6 feet per second and moves along the $x$-axis in such a way that at time $t$ its acceleration is $12t^2$ feet per second per second. Through how many feet does the particle move during the first 2 seconds?

(A) 16 ft

(B) 20 ft

(C) 24 ft

(D) 28 ft

(E) 32 ft

$A = 12t^2$
$V = \frac{12t^3}{3} + C = 4t^3 + C$
$6 = 0 + C$
$V = 4t^3 + 6$
$S = t^4 + 6t + C$
$C = 0$
$S = 2^4 + 6(2) = 28$

Ans

10. When the area of an expanding square, in square units, is increasing three times as fast as its side is increasing, in linear units, the side is

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) 3 (D) 2 (E) 1

$x$

$A = x^2$

$\frac{dA}{dt} = 3\frac{dx}{dt}$

$\frac{dA}{dt} = 3\frac{dx}{dt}$

$A = x^2$
$\frac{dA}{dt} = 2x\frac{dx}{dt}$

$\frac{dx}{dt}2x = 3\frac{dx}{dt}$

$x = \frac{3}{2}$

Ans

11. The average (mean) value of $\frac{1}{x}$ over the interval $1 \le x \le e$ is

(A) 1 (B) $\frac{1}{e}$ (C) $\frac{1}{e^2} - 1$ (D) $\frac{1+e}{2}$ (E) $\frac{1}{e-1}$

$\frac{1}{e-1}\int_1^e \frac{1}{x}\,dx$
$= \frac{1}{e-1}[\ln x]_1^e$
$= \frac{1}{e-1}(\ln e - \ln 1)$
$= \frac{1}{e-1}(1)$
$= \frac{1}{e-1}$

Ans

12. What is $\lim_{x \to \infty} \dfrac{3x^2 + 1}{(3 - x)(3 + x)}$?

(A) −9 (B) −3 (C) 1 (D) 3 (E) The limit does not exist.

Ans $\square$

---

13. If $\int_{-2}^{2} (x^7 + k)\, dx = 16$, then $k =$

(A) −12

(B) 12

(C) −4

(D) 4

(E) 0

Ans $\square$

---

14. Consider the function $f$ defined on $\frac{\pi}{2} \le x \le \frac{3\pi}{2}$ by $f(x) = \dfrac{\tan x}{\sin x}$ for all $x \neq \pi$. If $f$ is continuous at $x = \pi$, then $f(\pi) =$

(A) 2

(B) 1

(C) 0

(D) −1

(E) −2

$f(x) = \dfrac{\tan x}{\sin x} = \dfrac{\sin x}{\cos x} \cdot \dfrac{1}{\sin x} = \dfrac{1}{\cos x} = \dfrac{1}{\cos(\pi)} = -1$

$f(\pi) = \dfrac{\tan \pi}{\sin \pi} = \dfrac{0}{0}$

Ans $\square$

15. The function $f(x) = x^4 - 18x^2$ has a relative minimum at $x =$

(A) 0 and 3 only

(B) 0 and –3 only

(C) –3 and 3 only

(D) 0 only

(E) –3, 0, 3

Ans

16. The graph of $y = 3x^5 - 10x^4$ has an inflection point at

(A) (0, 0) and (2, –64)

(B) (0, 0) and (3, –81)

(C) (0, 0) only

(D) (–3, 81) only

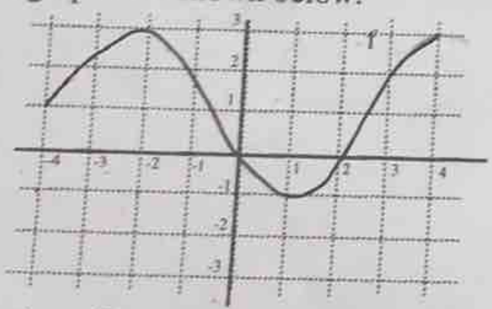
(E) (2, –64) only

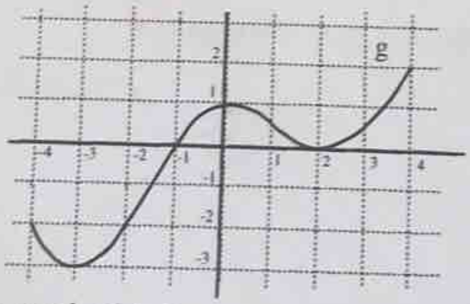
Ans

17. The composite function $h$ is defined by $h(x) = f[g(x)]$, where $f$ and $g$ are functions whose graphs are shown below.

$h'(x) = 0$

$f'(g(x)) \cdot g'(x)$




5

The number of horizontal tangent lines to the graph of $h$ is

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

$g'(x) = 0$

$x = -3, x = 1, x = 2$

$f'(g(x)) = 0$

$g(x) = -3, 1$

$x = -4, -2, x = 0$

3.5

Ans

18. The region in the first quadrant bounded by the graph of $y = \text{Arcsin}\, x$, $y = \frac{\pi}{2}$ and the $y$-axis, is rotated about the $y$-axis. The volume of the solid generated is given by

(A) $\pi \int_0^{\pi/2} y^2\, dy$ (B) $\pi \int_0^1 (\text{Arcsin}\, x)^2\, dx$ (C) $\pi \int_0^{\pi/2} (\text{Arcsin}\, x)^2\, dx$

(D) $\pi \int_0^{\pi/2} (\sin y)^2\, dy$ (E) $\pi \int_0^1 (\sin y)^2\, dy$

$y = \sin^{-1} x$
$x = \sin y$
$R(x) = x - 0 = \sin y$
$\int_0^{\pi/2} (\sin y)^2\, dy$
$y = \pi/2$

Ans

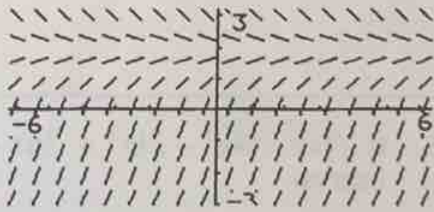
19. Find the coordinates of the absolute maximum point for the curve $y = xe^{-kx}$ where $k$ is a fixed positive number.

(A) $\left(\frac{1}{k}, \frac{1}{ke}\right)$ (B) $\left(\frac{-1}{k}, \frac{-e}{k}\right)$ (C) $\left(\frac{1}{k}, \frac{1}{e^k}\right)$ (D) $(0, 0)$ (E) there is no maximum

$y = xe^{-kx}$

Ans

20. The slope field for a differential equation $\frac{dy}{dx} = f(x, y)$ is given in the figure. The slope field corresponds to which of the following differential equations?




(A) $\frac{dy}{dx} = 2 - \ln x$

(B) $\frac{dy}{dx} = 2 - e^{-x}$

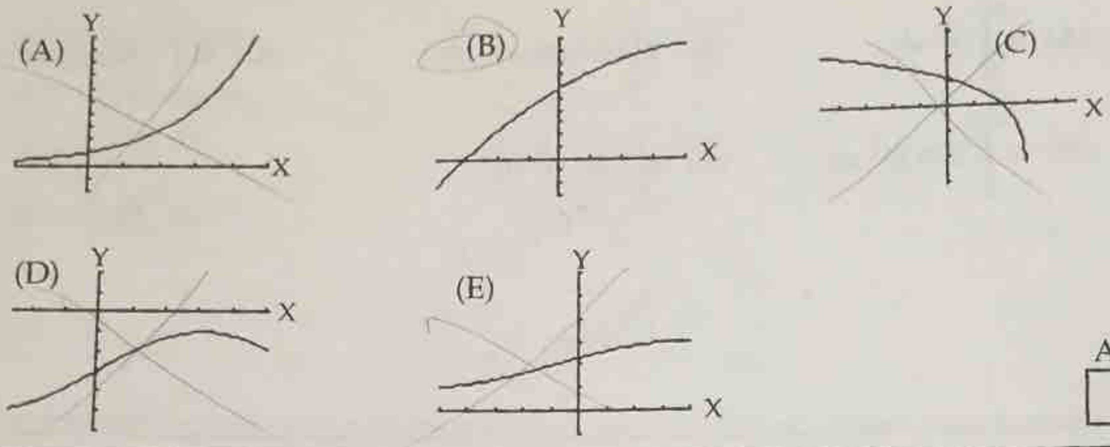
(C) $\frac{dy}{dx} = y - 2y^2$

(D) $\frac{dy}{dx} = 2 - y$

(E) $\frac{dy}{dx} = -x^2$

Ans

21. If $y$ is a function of $x$ such that $y' > 0$ for all $x$ and $y'' < 0$ for all $x$, which of the following could be part of the graph of $y = f(x)$?




Ans

22. Use the Trapezoid Rule with $n = 3$ to approximate the area under $y = x^2$ from $x = 1$ to $x = 4$.

(A) $\frac{45}{3}$

(B) $\frac{43}{3}$

(C) $\frac{43}{2}$

(D) 43

(E) 21

Ans

23. If $f(x) = 4x^3 - 21x^2 + 36x - 4$, then the graph of $f$ is decreasing and concave up on the interval

(A) $\left(\frac{3}{2}, 2\right)$ (B) $\left(-\infty, \frac{7}{4}\right)$ (C) $\left(\frac{7}{4}, \infty\right)$ (D) $\left(\frac{7}{4}, 2\right)$ (E) $\left(\frac{3}{2}, \frac{7}{4}\right)$

Ans

24. The number of bacteria in a culture is growing at a rate of $1500e^{3t/4}$ per unit of time $t$. At $t = 0$, the number of bacteria present was 2,000. Find the number present at $t = 4$.

(A) $2000e^3$

(B) $6000e^3$

(C) $2000e^6$

(D) $1500e^6$

(E) $1500e^3 + 500$

$y = 1500e^{3t/4}$

$\int 1500e^{3t/4} \quad \dfrac{1500e^{3t/4}}{3/4}$

$= 2000e^{3t/4} + c$

$2{,}000 = 2{,}000e^0 + c$

$c + 1 = 1$

$c = 0$

$2000e^{12/4} = 2000e^3$

Ans

25. A region in the plane is bounded by the graph of $y = \dfrac{1}{x}$, the $x$-axis, the line $x = m$ and the line $x = 3m$, $m > 0$. The area of this region

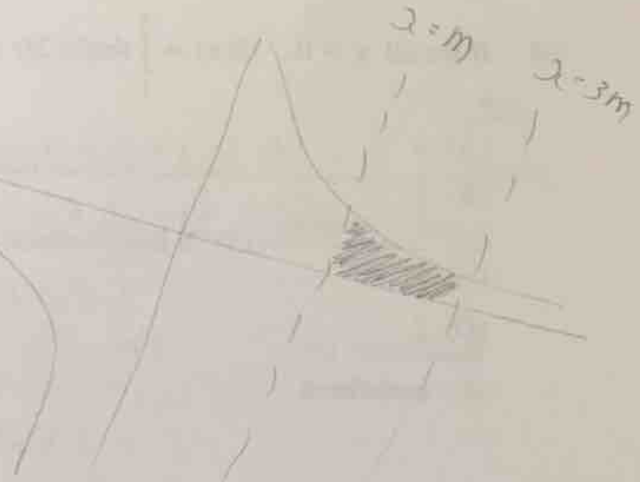
(A) is independent of $m$

(B) increases as $m$ increases

(C) decreases as $m$ increases

(D) decreases for all $m < \frac{1}{3}$

(E) increases for all $m < \frac{1}{3}$




$\int_m^{3m}$

Ans

26. The formula $x(t) = \ln t + \frac{t^2}{18} + 1$ gives the position of an object moving along the $x$-axis during the time interval $1 \le t \le 5$. At the instant when the acceleration of the object is zero, the velocity is

(A) 0 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) 1 (E) undefined

Ans

27. $\int 6 \sin x \cos^2 x \, dx =$

(A) $2 \sin^3 x + C$

(B) $-2 \sin^3 x + C$

(C) $2 \cos^3 x + C$

(D) $-2 \cos^3 x + C$

(E) $3 \sin^2 x \cos^2 x + C$

$-\frac{6\cos^3 x}{3} + C$

$= -2\cos^3 x + C$

Ans

28. If for all $x > 0$, $G(x) = \int_1^x \sin(\ln 2t)\, dt$, then the value of $G''\left(\frac{1}{2}\right)$ is

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) 2

(E) undefined

$G'(x) = \sin(\ln 2x)$

$G''(x) = \frac{2}{2x} \cos(\ln 2x)$

$= \frac{1}{x} \cos(\ln 2x)$

$G''(\frac{1}{2}) = 2\cos(\ln 1)$

$= 2\cos 0$

$= 2(1)$

$= 2$

Ans

EXAM IV
CALCULUS AB
SECTION I PART B
MULTIPLE-CHOICE
CALCULATORS
Time–50 minutes
Number of questions–17

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

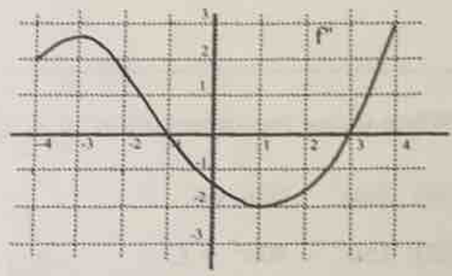
A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS IN THIS PART OF THE EXAMINATION.

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

1. The graph of the **second derivative** of a function $f$ is shown at the right. Which of the following is true?

the graph of $f''$




I. The graph of $f$ has an inflection point at $x = -1$. Yes

II. The graph of $f$ is concave down on the interval $(-1, 3)$. Yes

III. The graph of the derivative function $f'$ is increasing at $x = 1$. No

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, III

Ans

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2. If the function $f$ is continuous for all positive real numbers and if $f(x) = \dfrac{\ln x^2 - x\ln x}{x-2}$ when $x \neq 2$, then $f(2) =$

(A) $-1$ (B) $-2$ (C) $-e$ (D) $-\ln 2$ (E) undefined

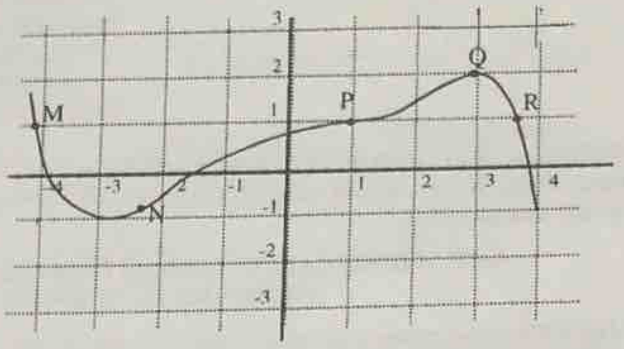
$$\frac{\ln 2^2 - 2\ln 2}{2-2}$$

Ans

3. The graph of the function $f$ is shown at the right. At which point on the graph of $f$ are all the following true?

$f(x) > 0$, and $f'(x) < 0$ and $f''(x) < 0$

the graph of $f$




(A) $M$ (B) $N$ (C) $P$ (D) $Q$ (E) $R$

Ans

4. When using the substitution $u = \sqrt{1+x}$, an antiderivative of $\int 60x\sqrt{1+x}\,dx$ is

$u^2 = 1+x$

$x = u^2 - 1$

(A) $20u^3 - 60u + C$

(B) $15u^4 - 30u^2 + C$

(C) $30u^4 - 60u^2 + C$

(D) $24u^5 - 40u^3 + C$

(E) $12u^6 - 20u^4 + C$

$u = \sqrt{1+x}$

$\frac{du}{dx} = \frac{1}{2}(1+x)^{-1/2}$

$2du = \frac{dx}{\sqrt{1+x}}$

$dx = 2\sqrt{1+x}\,du$

$\int 60x \cdot u \cdot 2\sqrt{1+x}\,du$

$= \int 120xu^2\,du$

$= \int 120(u^2-1)(u^2)\,du$

$= \int (120u^4 - 120u^2)\,du$

$= \frac{120u^5}{5} - \frac{120u^3}{3} + c$

$= 24u^5 - 40u^3 + c$

Ans

5. At $x = 0$, which of the following statements is TRUE of the function $f$ defined by $f(x) = \sqrt{x^2 + .0001}$.

I. $f$ is discontinuous II. $f$ has a horizontal tangent III. $f'$ is undefined

(A) I only (B) II only (C) III only (D) I and III only (E) I, II, III

$f'(x) = \frac{1}{2}(x^2 + .0001)^{-1/2}(2x)$

$= x(x^2 + .0001)^{-1/2}$

$= \frac{x}{\sqrt{x^2 + .0001}} = 0$ $x = 0$

Ans

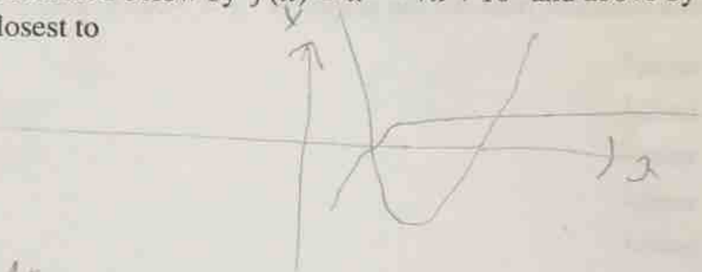
6. An object moves along the $x$-axis so that at time $t$, $t > 0$, its position is given by $x(t) = t^4 + t^3 - 30t^2 + 88t$. At the instant when the acceleration becomes zero, the velocity of the object is approximately

(A) 244 (B) 12 (C) 0 (D) –12 (E) –24

Ans

7. The area of the region bounded below by $f(x) = x^2 - 7x + 10$ and above by $g(x) = \ln(x-1)$ is closest to

(A) 7.35 units$^2$
(B) 7.36 units$^2$
(C) 7.38 units$^2$
(D) 7.40 units$^2$
(E) 7.42 units$^2$




Area $= \int_2^{5.134} [g(x) - f(x)]\,dx$
$=$

Ans

8. The average rate of change of the function $f(x) = \int_0^x \sqrt{1 + \cos(t^2)}\,dt$ over the interval $[1, 3]$ is nearest to

(A) 0.85
(B) 0.86
(C) 0.87
(D) 0.88
(E) 0.89

$\frac{f(3) - f(1)}{3-1} =$

Ans

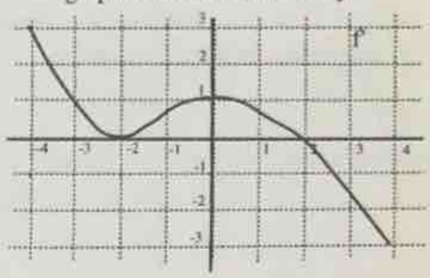
9. The graph of the *derivative* of $f$ is shown at the right. Which of the following is true about the function $f$?

I. $f$ is decreasing at $x = 0$. No

II. $f$ has a local maximum at $x = 2$. Yes

III. $f$ is concave up at $x = -1$. Yes

graph of the derivative of $f$




(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, III

Ans

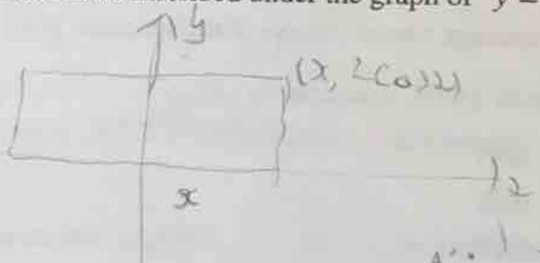
10. The total area enclosed between the graphs of $y = 3\cos x$ and $y = 1 - x$ is

(A) 4.92 $\text{units}^2$
(B) 4.94 $\text{units}^2$
(C) 4.96 $\text{units}^2$
(D) 4.98 $\text{units}^2$
(E) 5.00 $\text{units}^2$

Ans

11. What is the area of the largest rectangle that can be inscribed under the graph of $y = 2\cos x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$?

(A) 2.20 $\text{units}^2$
(B) 2.24 $\text{units}^2$
(C) 2.28 $\text{units}^2$
(D) 2.32 $\text{units}^2$
(E) 2.36 $\text{units}^2$




A':
-0.8 0.8

Area = 2x(2cos x)
= 4x cos x
A' = 4cos x + 4x(-sin x)
4cos x - 4x sin x = 0
4x sin x = 4cos x
x sin x = cos x

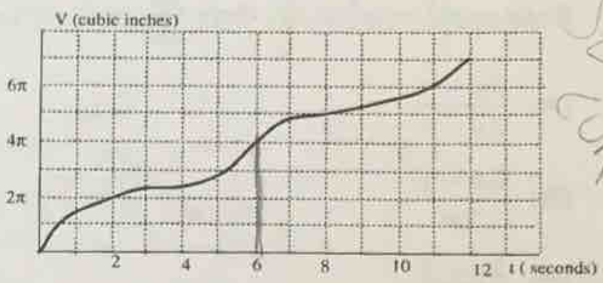
x tan x = 1
x = 0.86033, -0.86033
Largest rectangle:

Ans

12. The function $V$ whose graph is sketched below gives the volume of air, $V(t)$, (measured in cubic inches) that a man has blown into a balloon after $t$ seconds.

$$\left(V = \frac{4}{3}\pi r^3\right)$$

The rate at which the radius is changing after 6 seconds is nearest to




Slope curve

(A) 0.05 in/sec (B) 0.12 in/sec (C) 0.21 in/sec (D) 0.29 in/sec (E) 0.37 in/sec

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ $t=6$

$\pi = 4\pi(3^{2/3})\frac{dr}{dt}$

$1 = 4(3^{2/3})\frac{dr}{dt}$

$\frac{dr}{dt} = 0.12$

$\frac{4}{3}\pi r^3 = 4\pi$

$r^3 = 4 \cdot \frac{3}{4}$

$r^3 = 3$

$r = \sqrt[3]{3}$

$3'$

$\frac{dV}{dt} = \frac{V(7) - V(5)}{7-5}$

$= \frac{5\pi - 3\pi}{2}$

$= \frac{2\pi}{2} = \pi$

Ans

13. At how many points on the inteval $-2\pi \le x \le 2\pi$ does the tangent to the graph of the curve $y = x\cos x$ have slope $\frac{\pi}{2}$?

(A) 5

(B) 4

(C) 3

(D) 2

(E) 1

$y' = \cos x + x(-\sin x)$

$= -x\sin x + \cos x$

$-x\sin x + \cos x = \frac{\pi}{2}$

Ans

14. If $(x-y)^2 = y^2 - xy$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{2x-y}{2y-x}$

(B) $\dfrac{2x-y}{2x}$

(C) $\dfrac{2x-y}{x}$

(D) $\dfrac{2x+3y}{x}$

(E) undefined

$x^2 - 2xy + y^2 = y^2 - xy$

$x^2 - xy = 0$

$2x - (xy' + y) = 0$

$2x - xy' - y = 0$

$y' = \dfrac{y-2x}{-x}$

$= \dfrac{2x-y}{x}$

Ans

15. Let the base of a solid be the first quadrant region enclosed by the $x$-axis, the $y$-axis and the graph of $y = 1 - \dfrac{x^2}{4}$. If all the cross sections perpendicular to the $y$-axis are squares, then the volume of the solid is

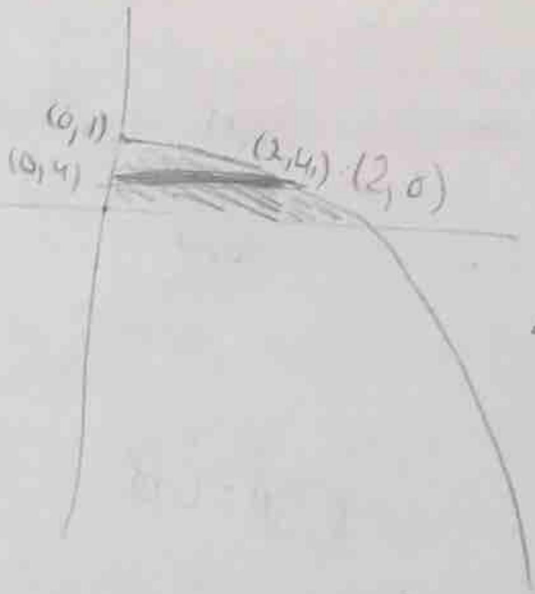
(A) 3 units$^3$

(B) 2 units$^3$

(C) 1 units$^3$

(D) $\frac{1}{2}$ units$^3$

(E) $\frac{1}{3}$ units$^3$




$R(x) = x - 0$
$= \sqrt{4-4y}$

$4y = 4 - x^2$

$x^2 = 4 - 4y$

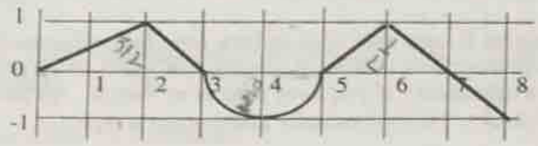
$x = \sqrt{4-4y}$


$\text{volume} = \int_0^1 \left(\sqrt{4-4y}\right)^2 dy$

Ans

16. Let the function $F$ be defined on the interval $[0, 8]$ by $F(x) = \int_0^x f(t)\, dt$, where the graph of $f$ is shown below.

$F(1) = 0$




graph of $y = f(t)$ $F'(2)$ or $f(2)$

In which of the following intervals does $F$ have a zero?

I. $4 < x < 5$ II. $5 < x < 6$ III. $6 < x < 7$

(A) I only (B) II only (C) III only (D) I and II only (E) I and III only

$\frac{1}{2}(3)(1) +$

$\frac{3}{2} - \frac{\pi(1)}{4}$

$=$

$\frac{1}{2}(3)(1) - \frac{4\pi}{2}$

$= \frac{3}{2} - 2\pi + \frac{1}{2}(2)(1)$

Ans

17. The change in $N$, the number of bacteria in a culture dish at time $t$, is given by: $\frac{dN}{dt} = 2N$. If $N = 3$ when $t = 0$, the approximate value of $t$ when $N = 1210$ is

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

$\frac{dN}{dt} = 2N$

$\frac{dN}{N} = 2dt$

$\ln N = 2t + c$

$\ln 3 = c$

$\ln N = 2t + \ln 3$

$\ln 1210 = 2t + \ln 3$

$t = \frac{\ln 1210 - \ln 3}{2}$

$t = 2.99989$

$\approx 3$

Ans

EXAM IV
CALCULUS AB
SECTION II, PART A
Time–45 minutes
Number of questions–3

A GRAPHING CALCULATOR IS REQUIRED ON THIS PART OF THE EXAMINATION.

- Before you begin Part A of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems and you will be able to come back to Part A (without a calculator), if you have time after Part B. All problems are given equal weight, but the parts of a particular solution are not necessarily given equal weight.

- You should write all work for each problem in the space provided. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed out work will not be graded.

- SHOW ALL YOUR WORK. You will be graded on the correctness and completeness of your methods as well as the accuracy of your final answers. Correct answers without supporting work may not receive full credit.

- Justifications require that you give mathematical (noncalculator) reasons and that you clearly identify functions, graphs, tables, or other objects you use.

- You are permitted to use your calculator in Part A to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate in your exam booklet the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

- Your work must be expressed in mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt($X^2$, $X$, 1, 5).

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

- Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

THE EXAM BEGINS ON THE NEXT PAGE

PLEASE TURN OVER

1. Let $y(t)$ be the temperature, in degrees Fahrenheit, of a cup of tea at time $t$ minutes, $t \geq 0$. Room temperature is $70^\circ$ and the initial temperature of the tea is $180^\circ$. The tea's temperature at time $t$ is described by the differential equation $\frac{dy}{dt} = -0.1(y - 70)$, with the initial condition $y(0) = 180$.

   (a) Use separation of variables to find an expression for $y$ in terms of $t$, where $t$ is measured in minutes.

   (b) How hot is the tea after 10 minutes?

   (c) If the tea is safe to drink when its temperature is less than $120^\circ$, at what time is the tea safe to drink?

$\frac{dy}{dt} = -0.1(y-70)$

$\frac{dy}{y-70} = -0.1dt$

$\ln|y-70| = -0.1t + C$

$y-70 = Ce^{-0.1t}$

$y(0) = 180$

$180-70 = Ce^{0}$

$C = 110$

$y-70 = 110e^{-0.1t}$

$y = 110e^{-0.1t} + 70$

b) $y = 110e^{-0.1(10)} + 70$
$= 110.467^\circ$

c) $110e^{-0.1t} + 70 < 120$

$110e^{-0.1t} < 50$

$e^{-0.1t} < \frac{5}{11}$ 4:

$-0.1t < \ln(\frac{5}{11})$ 7.885

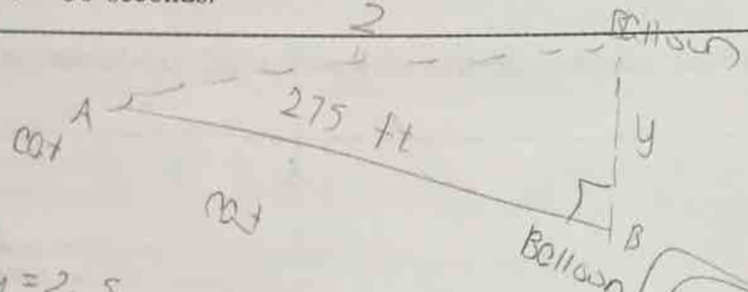
$-0.1t - \ln(\frac{5}{11}) < 0$

2. Two points, $A$ and $B$, are 275 ft apart. At a given instant, a balloon is released at $B$ and rises vertically at a constant rate of 2.5 ft/sec, and, at the same instant, a cat starts running from $A$ to $B$ at a constant rate of 5 ft/sec.

   (a) After 40 seconds, is the distance between the cat and the balloon decreasing or increasing? At what rate?

   (b) Describe what is happening to the distance between the cat and the balloon at $t = 50$ seconds.

1.471




$\frac{dy}{dt} = 2.5$ $\frac{dx}{dt} = 5$

$\boxed{RT = \text{Distance}}$

Distance =

$275^2 + y^2 = z^2$

$2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$RT = D = y$

$40 \cdot 2.5 = D$

$D = 100$

$2(100)(2.5) = 2\sqrt{85625}\frac{dz}{dt}$

$\frac{dz}{dt} = \frac{2y}{2z}\frac{dy}{dt}$

$\frac{dz}{dt} = \frac{y}{z}\frac{dy}{dt}$

$= \frac{100}{z}\frac{dy}{dt}$

$t = 40$

$275^2 + 100^2 = z^2$

$z = \sqrt{85625}$

$40 \cdot 5 =$

$2(275)(5) + 2(100)(2.5) = 2\sqrt{85625}\frac{dz}{dt}$

$1625 = \sqrt{85625}\frac{dz}{dt}$

$\frac{dz}{dt} =$

3. Suppose the derivative of a polynomial function $p$ is $p'(x) = (x+1)(x-1)(x-2)^2(x-4)^3$.

(a) What is the degree of $p$?

(b) What is the instantaneous rate of change of $p$ at $x = 6$?

(c) Find the intervals on which the graph of $p$ is increasing.

(d) Find the intervals on which the graph of $p$ is concave down.

$p'(x) = (x^2-1)(x-2)^2(x-4)^3$

$= (x^2-1)(x^2-2x+4)(x^2-8x+16)(x-4)$

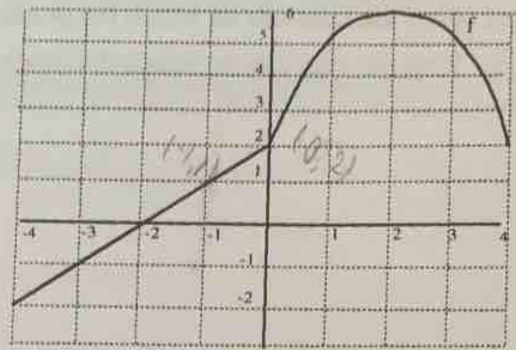
$= (x^4-2x^3+4x^2-x^2+(2x-4)(x^3-6x^2+16x-4x^2+32x-64)$

$= x^1$

A CALCULATOR MAY **NOT** BE USED ON THIS PART OF THE EXAMINATION.
DURING THIS 45 MINUTES YOU MAY GO BACK AND WORK ON PART A.

4. Answer the following questions about the function $f$, whose graph is shown at the right.

graph of the function $f$



(a) Find $\lim_{x \to 0} f(x)$. 2

/ (b) Find $\lim_{h \to 0^-} \frac{f(0+h)-f(0)}{h}$. $\frac{2-1}{0+1} = \frac{1}{-1}$

(c) Find $\lim_{x \to 0} f'(x)$.

(d) Find $\int_{-1}^{0} f(x)\,dx$. $\frac{1}{2}(1+2)(1) = \frac{3}{2}$ 3/2

(e) Approximate $\int_{-2}^{4} f(x)\,dx$ using the Trapezoid Rule with $n = 3$ subdivisions.

$\frac{4+2}{3}$ 2

$\frac{4+2}{2(3)}\left(f(-2)+2[f(0)+f(2)]+f(4)\right]$

$= \frac{6}{6}[0+2(2+6)+2]$

$= 1(16+2)$

$= 18$

$y = f(3)$

Slope at 0

$y = f(2)$

5. Let $g$ be the function given by $g(x) = \dfrac{x \cdot |x|}{x^2 + 1}$.

(a) Determine whether the *derivative* of the function $g$ is even, odd, or neither. odd

(b) Find $g'(2)$.

(c) Evaluate $\int_0^1 g(x)\,dx$.

(d) Determine $\lim_{x \to \infty} g(x)$.

(e) Find the range of $g$. Justify your answer.

$g(x) = \dfrac{x^2}{x^2+1}$, $x \geq 0$

$g(x) = \dfrac{-x^2}{x^2+1}$, $x < 0$

$g'(x) = \dfrac{(x^2+1)(2x) - x^2(2x)}{(x^2+1)^2}$, $x \geq 0$

$g'(x) = \dfrac{(x^2+1)(-2x) - (-x^2)(2x)}{(x^2+1)^2}$, $x < 0$

$g'(x) = \dfrac{2x}{(x^2+1)^2}$, $x \geq 0$

$g'(-x) = \dfrac{2(-x)}{(x^2+1)^2} = \dfrac{-2x}{(x^2+1)^2}$

$g'(-x) = -g'(x)$

$\therefore$ odd

$g'(x) = \dfrac{-2x}{(x^2+1)^2}$, $x < 0$

$g'(-x) = \dfrac{-2(-x)}{(x^2+1)^2} = \dfrac{2x}{(x^2+1)^2}$

$g'(-x) = -g'(x)$

$\lim_{x \to \infty} g(x) = 1$

$\lim_{x \to -\infty} g(x) = -1$

$-1 < x < 1$

$g'(2)$ $\dfrac{4}{5^2} = \dfrac{4}{25}$

$\int_0^1 \dfrac{-x^2}{x^2+1}\,dx$

$= \int_0^1$

$\tan\theta = x$

$\dfrac{dx}{d\theta} = \sec^2\theta$

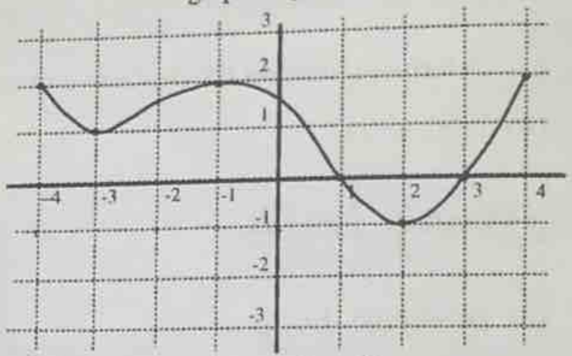
$\dfrac{-\tan^2\theta}{\sec^2\theta} \cdot \sec^2\theta$

$(1 - \sec^2\theta)$

$\dfrac{\sin^2\theta}{c}$

6. The graph of a differentiable function $f$ on the closed interval $-4, 4]$ is shown at the right.

graph of $f$




Let $G(x) = \int_{-4}^{x} f(t)\,dt$ for $-4 \le x \le 4$.

(a) Find $G(-4)$. $= 0$

(b) Find $G'(-1)$. $2$

(c) On which interval or intervals is the graph of $G$ concave down. Justify your answer.

(d) Find the value of $x$ at which $G$ has its maximum on the closed interval $[-4, 4]$. Justify your answer.

$G'(x) = f(x)$

$G'(-1) = f(-1) = 2$

c) $(-4,-3)$, $(-1,2)$

d) Rel max @ $x = 1$

$G(-4) = 0$

Max at $x = 1$ or endpoints

EXAM V
CALCULUS AB
SECTION I PART A
MULTIPLE CHOICE
NO CALCULATORS
Time–55 minutes
Number of questions–28

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

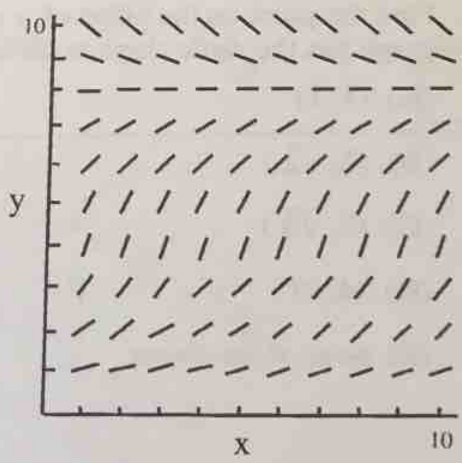
The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

1. If $y = \cos^2(2x)$, then $\frac{dy}{dx} =$

(A) $2\cos 2x \sin 2x$
(B) $-4\sin 2x \cos 2x$
(C) $2\cos 2x$
(D) $-2\cos 2x$
(E) $4\cos 2x$

Ans

2. A slope field for a differential equation $\frac{dy}{dx} = f(x,y)$ is given in the figure at the right. Which of the following statements are true?




I. The value of $\frac{dy}{dx}$ at the point $(2, 2)$ is approximately 1.

II. As $y$ approaches 8 the rate of change of $y$ approaches zero.

III. All solution curves for the differential equation have the same slope for a given value of $x$.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, III

Ans

3. The slope of the line tangent to the graph of $y = \ln\sqrt{x}$ at $(e^2, 1)$ is

(A) $\frac{e^2}{2}$ (B) $\frac{2}{e^2}$ (C) $\frac{1}{2e^2}$ (D) $\frac{1}{2e}$ (E) $\frac{1}{e}$

Ans

322 4. Which of the following functions is both continuous and differentiable at all $x$ in the interval $-2 \le x \le 2$?

$x^2-1, \; x<-1, \; x>1$ No
$1-x^2, \; -1 \le x < 1$

$x^2-1 \ge 0$
$(x+1)(x-1) \ge 0$

(A) $f(x) = |x^2 - 1|$

(B) $f(x) = \sqrt{x^2 - 1}$ $f'(x) = \frac{1}{2}(x^2-1)^{-1/2}(2x)$ $= \frac{x}{\sqrt{x^2-1}}$

(C) $f(x) = \sqrt{x^2 + 1}$

(D) $f(x) = \frac{1}{x^2 - 1}$ $x^2-1 \ge 0$ $x \le -1, x \ge 1$ $\frac{f(x)-f(c)}{x-c}$ or $f'(x)$ for

(E) none of these

$\frac{1}{2}(x^2+1)^{-1/2}(2x)$
$= \frac{x}{\sqrt{x^2+1}}$ $x^2+1 \ge 0$

Ans

5. Find the point on the graph of $y = \sqrt{x}$ between (1, 1) and (9, 3) at which the tangent to the graph has the same slope as the line through (1, 1) and (9, 3).

(A) (1, 1)

(B) $(2, \sqrt{2})$

(C) $(3, \sqrt{3})$

(D) (4, 2)

(E) none of the above

$\frac{3-1}{9-1} = \frac{2}{8} = \frac{1}{4}$

$y' = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$\frac{1}{2\sqrt{x}} = \frac{1}{4}$

$2\sqrt{x} = 4$

$\sqrt{x} = 2$

$x = 2$

Ans

6. Consider the function $f(x) = \dfrac{x^4}{2} - \dfrac{x^5}{10}$. The *derivative* of $f$ attains its maximum value at $x =$

(A) 3

(B) 4

(C) 5

(D) 0

(E) there is no maximum

$f'(x) = 2x^3 - \frac{1}{2}x^4$

$f''(x) = 6x^2 - 2x^3 = 0$

$2x^2(3 - x) = 0$

$x = 0 \quad x = 3$

$f''(x)$ − + −

0 3

max

Ans

7. The acceleration, $a(t)$, of a body moving in a straight line is given in terms of time $t$ by $a(t) = 4 - 6t$. If the velocity of the body is 20 at $t = 0$ and if $s(t)$ is the distance of the body from the origin at time $t$, what is $s(3) - s(1)$?

(A) −10

(B) 0

(C) 10

(D) 20

(E) 30

$\cancel{\frac{s(3) - s(1)}{3-1} =}$

$v(t) = \int (4 - 6t)\,dt$

$= 4t - 3t^2 + C$

$20 = 0 - 0 + C$

$C = 20$

$v(t) = 4t - 3t^2 + 20$

$s(t) = \int (4t - 3t^2 + 20)\,dt$

$= 2t^2 - t^3 + 20t + C$

$0 = C$

$s(t) = 2t^2 - t^3 + 20t$

$s(3) = 2(9) - 27 + 60$

$= 51$

$s(1) = 2 - 1 + 20 = 21$

$s(3) - s(1) = 51 - 21 = 30$

Ans

8. $\lim\limits_{x \to 1} \left( \dfrac{\sqrt{x+3} - 2}{1 - x} \right)$

(A) 0.5

(B) 0.25

(C) 0

(D) −0.25

(E) −0.5

$\lim\limits_{x \to 1} \left( \dfrac{\sqrt{x+3} - 2}{1 - x} \cdot \dfrac{\sqrt{x+3} + 2}{\sqrt{x+3} + 2} \right)$

$\lim\limits_{x \to 1} = \dfrac{x + 3 - 4}{(1 - x)(\sqrt{x+3} + 2)}$

$\lim\limits_{x \to 1} = \dfrac{x - 1}{(1 - x)(\sqrt{x+3} + 2)}$

$= \dfrac{-1}{\sqrt{x+3} + 2}$

$= \dfrac{-1}{2+2} = \dfrac{-1}{4}$

Ans

9. Let $f$ be defined by $f(x) = \begin{cases} \dfrac{x^2 - 2x + 1}{x - 1} & \text{for } x \neq 1 \\ k & \text{for } x = 1. \end{cases}$

Determine the value of $k$ for which $f$ is continuous for all real $x$.

$\lim_{x \to 1} \frac{(x-1)^2}{x-1} = \lim_{x \to 1} x - 1 = 0 = k$

(A) 0

(B) 1

(C) 2

(D) 3

(E) none of the above

Ans

10. The average value of $f(x) = e^{2x} + 1$ on the interval $0 \le x \le \frac{1}{2}$ is

(A) $e$ (B) $\frac{e}{2}$ (C) $\frac{e}{4}$ (D) $2e - 1$ (E) $\frac{e^{2x} + 1}{2}$

Ans

11. A point moves on the $x$-axis in such a way that its velocity at time $t > 0$ is given by $v = \dfrac{e^t}{t}$.
At what value of $t$ does $v$ attain its minimum?

(A) 0

(B) 1

(C) $e$

(D) $-1$

(E) There is no minimum value of $v$.

$v = \frac{e^t}{t}$

$\frac{dv}{dt} = \frac{t(e^t) - e^t(1)}{t^2}$

$= \frac{te^t - e^t}{t^2} = 0$

$e^t(t-1) = 0 \quad t \neq 0$

$t = 1$

$v'$: $-$ $+$

0 1

$1 - 1$

Ans

12. $\int \frac{4x}{1+x^2}\,dx =$

(A) $4\text{Arctan}\,x + C$ (B) $\frac{4}{x}\text{Arctan}\,x + C$ (C) $\frac{1}{2}\ln(1+x^2) + C$

(D) $2\ln(1+x^2) + C$ (E) $2x^2 + 4\ln|x| + C$

$\frac{4\ln(1+x^2)}{2} = 2\ln(1+x^2)$

Ans

13. Let $f(x) = x^4 + ax^2 + b$. The graph of $f$ has a relative maximum at $(0, 1)$ and an inflection point when $x = 1$. The values of $a$ and $b$ are

(A) $a = 1, \quad b = -6$

(B) $a = 1, \quad b = 6$

(C) $a = -6, \quad b = 5$

(D) $a = -6, \quad b = 1$

(E) $a = 6, \quad b = 1$

$x^4 - 6x^2 + b = f(x)$

$b = 1$

$f'(x) = 4x^3 + 2ax = 0$

$2x(2x^2 + a) = 0$

$x = 0$

$2x^2 + a = 0$

$x^2 = -\frac{a}{2}$

$f''(x) = 12x^2 + 12a = 0$

$12(1) + 2a = 0$

$2a = -12$

$a = -6$

Ans

14. $\int_1^2 \frac{x^2 - x}{x^3}\,dx =$

(A) $\ln 2 - \frac{1}{2}$ (B) $\ln 2 + \frac{1}{2}$ (C) $\frac{1}{2}$ (D) 0 (E) $\frac{1}{4}$

$x^{-2}$

$\int_1^2 \frac{1}{x} - \frac{1}{x^2}$

$\left[\ln x + \frac{1}{x}\right]_1^2$

$= \ln 2 + \frac{1}{2} - \ln 1 - 1$

$= \ln 2 - \frac{1}{2}$

Ans

15. The edge of a cube is increasing at the uniform rate of 0.2 inches per second. At the instant when the total surface area becomes 150 square inches, what is the rate of increase, in cubic inches per second, of the volume of the cube?

(A) 5 $in^3/sec$

(B) 10 $in^3/sec$

(C) 15 $in^3/sec$

(D) 20 $in^3/sec$

(E) 25 $in^3/sec$

$6x^2 = 150$

$x^2 = 25$

$x = 5$

$\frac{dx}{dt} = 0.02$

$V = x^3$

$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

$= 3(5)^2(0.02)$

$= 25 \cdot 0.06$

Ans

16. $\int_0^{\sqrt{3}} \frac{x\,dx}{\sqrt{1+x^2}} =$

(A) $\frac{1}{2}$

(B) 1

(C) 2

(D) ln 2

(E) Arctan $2 - \frac{\pi}{4}$

$\int_0^{\sqrt{3}} x(1+x^2)^{-1/2}\,dx$

$\frac{(1+x^2)^{1/2}}{2 \cdot 1/2} = (1+x^2)^{1/2}$

$= \left[\sqrt{1+x^2}\right]_0^{\sqrt{3}}$

$= \sqrt{1+3} - \sqrt{1} = 2 - 1 = 1$

Ans

17. Which of the following is true about the graph of $f(x) = \ln\left|x^2 - 4\right|$ in the interval $(-2, 2)$ ?

(A) $f$ is increasing.

(B) $f$ attains a relative minimum at (0, 0).

(C) $f$ has a range of all real numbers.

(D) $f$ is concave down.

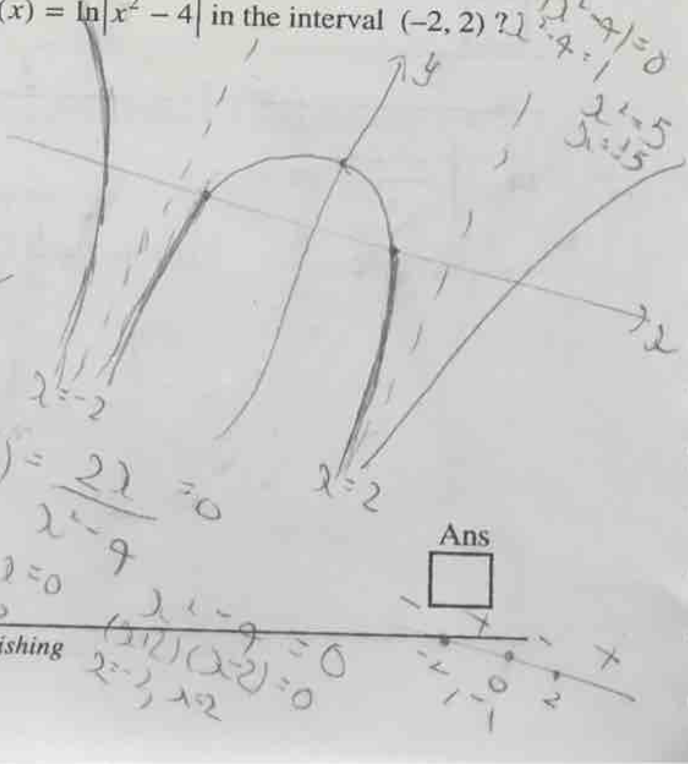
(E) $f$ has an asymptote at $x = 0$.

$x^2 - 4 \neq 0$

$x^2 \neq 4$

$x \neq \pm 2$

Look at graph



$\lim_{x\to\infty} \ln|x^2-4| = \ln\infty = \infty$

$\lim_{x\to-\infty} = \ln\infty = \infty$

$\lim_{x\to 2^+} \ln|x^2-4| = \infty$

$\lim_{x\to 2^-} |x^2-4| = \infty$

$f'(x) = \frac{2x}{x^2-4} = 0$

$2x = 0$

$x = 0$

$x^2 - 4 = 0$

$(x+2)(x-2) = 0$

$x = -2,\ x = 2$

Ans

18. If $g(x) = \text{Arcsin } 2x$, then $g'(x) =$

(A) $2\text{Arccos } 2x$ (B) $2\csc 2x \cot 2x$ (C) $\dfrac{2}{1+4x^2}$

(D) $\dfrac{2}{\sqrt{4x^2-1}}$ (E) $\dfrac{2}{\sqrt{1-4x^2}}$

$g(x) = \sin^{-1} 2x$
$g'(x) = \dfrac{2}{\sqrt{1-(2x)^2}} = \dfrac{2}{\sqrt{1-4x^2}}$

Ans

---

19. $\int x(x^2-1)^4 \, dx =$

(A) $\frac{1}{10}(x^2)(x^2-1)^5 + C$

(B) $\frac{1}{10}(x^2-1)^5 + C$

(C) $\frac{1}{5}(x^3-x)^5 + C$

(D) $\frac{1}{5}(x^2-1)^5 + C$

(E) $\frac{1}{5}(x^2-x)^5 + C$

$\int x(x^2-1)^4 dx$
$= \dfrac{(x^2-1)^5}{5 \cdot 2} + c$
$= \dfrac{1}{10}(x^2-1)^5 + c$

Ans

---

20. If $y = e^{kx}$, then $\dfrac{d^5y}{dx^5} =$

(A) $k^5 e^x$

(B) $k^5 e^{kx}$

(C) $5! e^{kx}$

(D) $5! e^x$

(E) $5e^{kx}$

$y = e^{kx}$
$\dfrac{dy}{dx} = ke^{kx}$
$\dfrac{d^2y}{dx^2} = k^2 e^{kx}$

Ans

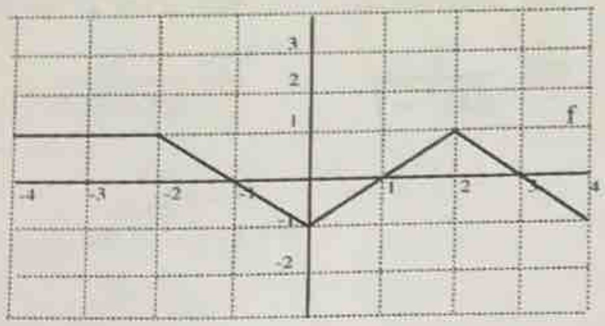
21. The graph of $f$ is shown at the right. Which of the following statements are true?

I. $f(2) > f'(1)$ No

II. $\int_0^1 f(x)\,dx > f'(3.5)$ yes

III. $\int_0^1 f(x)\,dx > \int_0^2 f(x)\,dx$ No

the graph of $f$




$-\frac{1}{2}(2)(1) = -1$ $\quad -\frac{1}{2}(1)(1) + \frac{1}{2}(1)(1) = -\frac{1}{2}+\frac{1}{2}$

$\frac{f(4)-f(3)}{4-3} = \frac{-1-0}{1} = -1$

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, III

$f'(1) = \frac{1+1}{2-0} = \frac{2}{2} = 1$ $\quad \int_0^1 f(x) = -\frac{1}{2}(1)(1) = -\frac{1}{2}$

Ans

22. If $g(x) = \sqrt{x}\,(x-1)^{2/3}$, then the domain of $g'$ is

(A) $\{x \mid 0 < x\}$

(B) $\{x \mid \cancel{x \neq 0}$ and $x \neq 1\}$ $x > 0$

(C) $\{x \mid 0 < x < 1$ or $x > 1\}$

(D) $\{x \mid 0 < x < 1\}$

(E) $\{x \mid$ all real numbers$\}$

$g' = \frac{1}{2}x^{-1/2}(x-1)^{2/3} + \sqrt{x}\cdot\frac{2}{3}(x-1)^{-1/3}$

$= \frac{(x-1)^{2/3}}{2\sqrt{x}} + \frac{2\sqrt{x}}{3(x-1)^{1/3}}$

$= \frac{3(x-1)^{2/3}(x-1)^{1/3} + 4\sqrt{x}(\sqrt{x})}{6\sqrt{x}(x-1)^{1/3}}$

$x > 0 \quad x-1 \neq 0 \quad x \neq 1$

Ans

23. A point moves on the $x$-axis so that its distance from the origin at time $t$ is given by $10t - 4t^2$. What is the *total* distance covered by the point between $t = 1$ and $t = 2$?

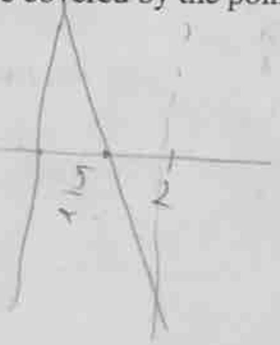
(A) 1.0

(B) 1.5

(C) 2.0

(D) 2.5

(E) 3.0




$S(t) = 10t - 4t^2$

$V(t) = 10 - 8t$

total distance:

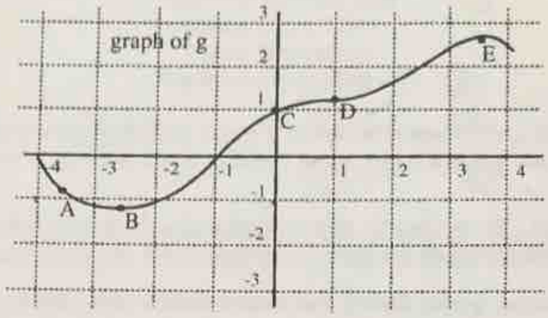
$\int_1^{5/4} v(t) - \int_{5/4}^2 v(t)$

$= \left[10t - 4t^2\right]_1^{5/4} - \left[10t - 4t^2\right]_{5/4}^2$

$= 2.5$

Ans

24. At which point on the graph of $y = g(x)$ below is $g'(x) = 0$ and $g''(x) = 0$?




(A) A (B) B (C) C (D) D (E) E

Ans

25. If $y$ is a differentiable function of $x$, then the slope of the tangent to the curve $xy - 2y + 4y^2 = 6$ at the point where $y = 1$ is

(A) $\frac{1}{12}$ (B) $-\frac{1}{10}$ (C) $-\frac{1}{6}$ (D) $\frac{1}{4}$ (E) $-\frac{5}{6}$

Ans

26. The area of the region bounded above by $y = 1 + \sec^2 x$, below by $y = 0$, on the left by $x = 0$ and on the right by $x = \frac{\pi}{4}$ is approximately

(A) 1 units$^2$ (B) 1.25 units$^2$ (C) 1.5 units$^2$ (D) 1.75 units$^2$ (E) 2 units$^2$

$$\int_0^{\pi/4} (1+\sec^2 x)\,dx$$
$$= \left[x + \tan x\right]_0^{\pi/4}$$
$$= \frac{\pi}{4} + \tan\frac{\pi}{4} - \tan 0$$
$$= \frac{\pi}{4} + 1 - 0 = \frac{\pi}{4} + 1$$

Ans

27. A solution of the equation $\dfrac{dy}{dx} + 2xy = 0$ that contains the point $(0, e)$ is

(A) $y = e^{1-x^2}$

(B) $y = e^{1+x^2}$

(C) $y = e^{1-x}$

(D) $y = e^{1+x}$

(E) $y = e^{x^2}$

$\frac{dy}{dx} = -2xy$

$\frac{dy}{y} = -2x\,dx$

$\ln y = -x^2 + c$

$y = Ce^{-x^2}$

$(0, e)$

$e = Ce^0$

$C = e$

$y = e\,e^{-x^2}$

$= e^{1-x^2}$

Ans $\square$

28. Which of the following are true about the function $F(x) = \int_1^x \ln(2t - 1)\, dt$ ?

yes yes

I. $F(1) = 0$ II. $F'(1) = 0$ $F''(1) = 1$

(A) I and II only

(B) I and III only

(C) II and III only

(D) I, II, III

(E) none

$F'(x) = \ln(2x-1)$

$F'(1) = \ln 1 = 0$

$F''(x) = \frac{2}{2x-1}$

$F''(1) = \frac{2}{1} = 2$

Ans $\square$

EXAM V
CALCULUS AB
SECTION I PART B
MULTIPLE-CHOICE
CALCULATORS
Time–50 minutes
Number of questions–17

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS IN THIS PART OF THE EXAMINATION.

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
(2) Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

1. How many points of inflection does the graph of $y = \cos x + \frac{1}{3}\cos 3x - \frac{1}{5}\cos 5x$ have on the interval $0 \le x \le \pi$ ?

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

Ans $\boxed{\phantom{x}}$

2. Oil is leaking from a tanker at the rate of $R(t) = 500e^{-0.2t}$ gallons per hour, where $t$ is measured in hours. The amount of oil that has leaked out after 10 hours is closest to

$\int_0^{10} 500e^{-0.2t}$

(A) 2140 gals
(B) 2150 gals
(C) 2160 gals
(D) 2170 gals
(E) 2180 gals

Ans $\boxed{\phantom{x}}$

3. The sale of lumber $S$ (in millions of square feet) for the years 1980 to 1990 is modeled by the function

$$S(t) = 0.46\cos(0.45t + 3.15) + 3.4$$

where $t$ is the time in years with $t = 0$ corresponding to the beginning of 1980. Determine the year when lumber sales were increasing at the greatest rate.

$t = 3$

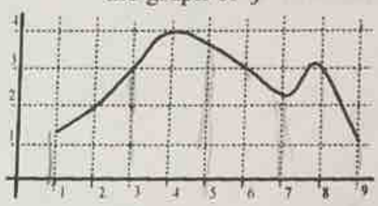
(A) 1982
(B) 1983
(C) 1984
(D) 1985
(E) 1986

$S'(t) = -0.46(0.45)\sin(0.45t+3.15)$
$= -0.207\sin(0.45t+3.15)$
$S''(t) = -0.09315\cos(0.45t+3.15) = 0$
$t = 3.472$
$\cos(0.45t+3.15) = 0$

Ans

4. The graph of $f$ over the interval [1, 9] is shown in the figure. Using the data in the figure, find a midpoint approximation with 4 equal subdivisions for $\int_1^9 f(x)\,dx$.

the graph of $f$



(A) 20 (B) 21 (C) 22 (D) 23 (E) 24

$= \frac{9-1}{4}\left[f(2)+f(4)+f(6)+f(8)\right]$
$= 2(2+4+3+3) = 2(12) = 24$

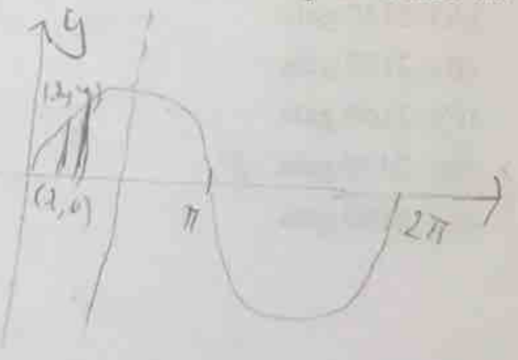
Ans

5. Let the base of a solid be the first quadrant region enclosed by the $x$-axis and one arch of the graph of $y = \sin x$. If all cross sections perpendicular to the $x$-axis are squares, then the volume of the solid is approximately

$R(x) = \sin x - 0$
$= \sin x$

(A) 0.52 units$^3$
(B) 0.79 units$^3$
(C) 1.05 units$^3$
(D) 1.57 units$^3$
(E) 2.00 units$^3$

sin x




Volume $= \int_0^{\pi} (\sin x)^2\,dx = 1.571$

Ans

6. If $f(x) = 2x + \sin x$ and the function $g$ is the inverse of $f$, then $g'(2) =$

(A) 0.32
(B) 0.34
(C) 0.36
(D) 0.38
(E) 0.40

$2x + \sin x \to \boxed{f} \to y = 2$

$x = 2 \to \boxed{f^{-1}}$

$2x + \sin x = 2$

$x = 0.68403666$

$y = 2.775x + c$

$x = 2.775y + c$

$y = \frac{x - c}{2.775}$

$f^{-1} = g(x)$

$f'(x) = 2 + \cos x$

$g'(2) = \frac{1}{f'g(x)} = \frac{1}{f'g(2)} = \frac{1}{f'(0.684)}$

$2 = 2x + \sin x$

Ans

7. Administrators at Massachusetts General Hospital believe that the hospital's expenditures $E(B)$, measured in dollars, are a function of how many beds $B$ are in use with

$$E(B) = 14000 + (B+1)^2.$$

On the other hand, the number of beds $B$ is a function of time $t$, measured in days, and it is estimated that

$$B(t) = 20\sin\left(\frac{t}{10}\right) + 50.$$

$= 20\sin(10) + 50$
$= 39.119$

At what rate are the expenditures decreasing when $t = 100$?

(A) 120 dollars/day
(B) 125 dollars/day
(C) 130 dollars/day
(D) 135 dollars/day
(E) 140 dollars/day

$E'(B) = 2(B+1) \cdot \frac{dB}{dt}$

$B(t) = 20\sin\left(\frac{t}{10}\right) + 50$

$B'(t) = \frac{20}{10}\cos\left(\frac{t}{10}\right)$

$= 2\cos\left(\frac{t}{10}\right)$

When $t = 100$

$B(100) = 20\sin 10 + 50$

$E'(B) = 2(B+1) \cdot 2\cos\left(\frac{t}{10}\right)$

$= 2[20\sin 10 + 51] \cdot 2\cos\left(\frac{t}{10}\right)$

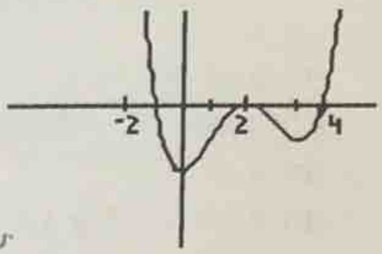
$= 2[20\sin 10 + 51] \cdot 2\cos 10$

$= -134.653$

Ans

8. Let $f$ be a function that has domain $[-2, 5]$. The graph of $f'$ is shown at the right. Which of the following statements are TRUE?

The graph of $f'$




I. $f$ has a relative maximum at $x = -1$. Y

II. $f$ has an absolute minimum at $x = 0$. NO

III. $f$ is concave down for $-2 < x < 0$. Y

IV. $f$ has inflection points at $x = 0$ and $x = 2$ and $x = 3$. Y

(A) I, II, IV (B) I, III, IV (C) II, III, IV (D) I, II, III (E) I, II, III, IV

Ans

9. On which interval is the graph of $f(x) = 4x^{3/2} - 3x^2$ both concave down and increasing?

(A) $(0, 1)$

(B) $\left(0, \frac{1}{2}\right)$

(C) $\left(0, \frac{1}{4}\right)$

(D) $\left(\frac{1}{4}, \frac{1}{2}\right)$

(E) $\left(\frac{1}{4}, 1\right)$

Ans

10. If the graph of $y = f(x)$ is defined for all $x \geq 0$, contains the point $(0, 1)$, has $\frac{dy}{dx} = 3\sqrt{xy}$ and $f(x) > 0$ for all $x$, then $f(x) =$

(A) $(x^3 + 1)^2$

(B) $(x^{3/2} + 1)^2$

(C) $(x^{1/2} + 1)^2$

(D) $(x^3 - 1)^2$

(E) $(x^{3/2} - 1)^2$

Ans

11. If $\sin 3x - 1 = \int_a^x f(t)\,dt$, then the value of $a$ is

(A) 0

(B) 1

(C) $-1$

(D) $\frac{\pi}{3}$

(E) $\frac{\pi}{6}$

$3\cos 3x = f(x)$

$\sin 3x - 1 = \int_a^x 3\cos 3x$

$\sin 3x - 1 = [\sin 3x]_a^x$

$\sin 3x - 1 = \sin 3x - \sin 3a$

$-\sin 3a = -1$

$\sin 3a = 1$

$3a = \frac{\pi}{2}$

$a = \frac{\pi}{6}$

Ans $\square$

12. If $xy^2 = 20$ and $x$ is decreasing at the rate of 3 units per second, the rate at which $y$ is changing when $y = 2$ is nearest to

(A) $-0.6$ units/sec

(B) $-0.2$ units/sec

(C) 0.2 units/sec

(D) 0.6 units/sec

(E) 1.0 units/sec

$xy^2 = 20$

$\frac{dx}{dt} = -3$

$\frac{dx}{dt}y^2 + 2(xy)\frac{dy}{dt} = 0$

$(-3)(4) + 5(4)\frac{dy}{dt} = 0$

$x(4) = 20$

$x = 5$

$\frac{dy}{dt} = \frac{12}{20} = 0.6$

Ans $\square$

13. An approximation for $\int_{-1}^{2} e^{\sin(1.5x-1)}\,dx$ using a right-hand Riemann sum with three equal subdivisions is nearest to

(A) 2.5

(B) 3.5

(C) 4.5

(D) 5.5

(E) 6.5

$\frac{2+1}{3}\left[f(0) + f(1) + f(2)\right]$

$\left[e^{\sin(-1)} + e^{\sin(0.5)} + e^{\sin 2}\right]$

Ans $\square$

14. If $f(x)$ is defined on $-\pi \le x \le \pi$ and $\frac{dy}{dx} = \frac{\cos x}{x^2+1}$, which of the following statements about the graph of $y = f(x)$ is true?

(A) The graph has no relative extremum.
(B) The graph has one point of inflection and two relative extrema.
(C) The graph has two points of inflection and one relative extremum.
(D) The graph has two points of inflection and two relative extrema.
(E) The graph has three points of inflection and two relative extrema.

$\frac{\cos x}{x^2+1} = 0$ $\quad x^2+1=0$

$\cos x = 0$

$x = -\frac{\pi}{2}, \frac{\pi}{2}$

2 extrema

$f'(x)$: $-$ $+$ $-$ at $-\pi, -\frac{\pi}{2}, \frac{\pi}{2}, \pi$

$f''(x) = \frac{(x^2+1)(-\sin x) - \cos x(2x)}{(x^2+1)^2}$

$= \frac{-x^2\sin x - \sin x - 2x\cos x}{(x^2+1)^2}$

$-x^2\sin x - \sin x - 2x\cos x = 0$

$\sin x(-x^2-1) - 2x\cos x = 0$

$\sin x(-x^2-1) = 2x\cos x$

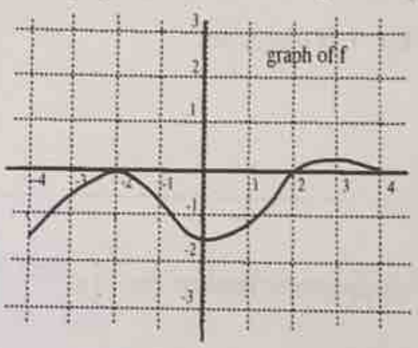
$\tan x = \frac{-2x}{x^2+1}$

3 inf

Ans

15. The graph of the function $f$ is shown at the right. If the function $G$ is defined by $G(x) = \int_{-4}^{x} f(t)\,dt$, for $-4 \le x \le 4$, which of the following statements about $G$ are true?

graph of the function $f$ $\quad G'(x)$




I. $G$ is increasing on $(1, 2)$. No
II. $G$ is decreasing on $(-4, -3)$. Yes
III. $G(0) < 0$. Yes

(A) None (B) II only (C) III only (D) II and III only (E) I and II only

$G(0) = \int_{-4}^{0} f(t)\,dt$

Ans

16. The function $f$ is defined on all the reals such that $f(x) = \begin{cases} x^2 + kx - 3 & \text{for } x \le 1 \\ 3x + b & \text{for } x > 1. \end{cases}$

For which of the following values of $k$ and $b$ will the function $f$ be both continuous and differentiable on its entire domain?

(A) $k = -1,\ b = -3$

(B) $k = 1,\ b = 3$

(C) $k = 1,\ b = 4$

(D) $k = 1,\ b = -4$

(E) $k = -1,\ b = 6$

$\lim_{x\to1^+} 3x+3 = \lim_{x\to1^-} x^2+kx-3$

$3+b = 1-3+k$

$\boxed{3+b = k-2}$

$\lim_{x\to1^+} \frac{3x+b-(1+k-3)}{x-1} = \lim_{x\to1^-} \frac{x^2+kx-3-(1+k-3)}{x-3}$

$\lim_{x\to1^+} \frac{3x+b+2-k}{x-1} = \lim_{x\to1^-} \frac{x^2+kx-1-k}{x-1}$

$\lim_{x\to1^+} \frac{3x+k-5+2-k}{x-1} = \lim_{x\to1^-} \frac{x^2+k(x-1)-1}{x-1}$

$\lim_{x\to1^+} \frac{3x-3}{x-1} = \lim_{x\to1^-} \frac{(x-1)(x+1)+k(x-1)}{x-1}$

$\lim_{x\to1^+} \frac{3(x-1)}{x-1} = \lim_{x\to1^-} x+1+k$

$\lim_{x\to1^+} = 3 \qquad = k+2$

$k+2 = 3$

$k = 1 \qquad b = 1-5 = -4$

Ans

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17. A particle moves along the $x$-axis with velocity at time $t$ given by: $v(t) = t + 2\sin t$. If the particle is at the origin when $t = 0$, its position at the time when $v = 6$ is $x =$

(A) 17.14 (B) 19.16 (C) 23.18 (D) 29.20 (E) 39.30

$V(t) = t + 2\sin x$

$S(t) = \int t + 2\sin t$

$= \frac{t^2}{2} - 2\cos t + C$

$0 = 0 - 2\cos(0) + C$

$0 = -2(1) + C$

$C = 2$

when $v = 6$, $x = 6.1786965$

$S(t) = \frac{t^2}{2} - 2\cos t + 2$

$t + 2\sin t = 6$

Ans

EXAM V
CALCULUS AB
SECTION II, PART A
Time–45 minutes
Number of questions–3

A GRAPHING CALCULATOR IS REQUIRED ON THIS PART OF THE EXAMINATION.

- Before you begin Part A of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems and you will be able to come back to Part A (without a calculator), if you have time after Part B. All problems are given equal weight, but the parts of a particular solution are not necessarily given equal weight.

- You should write all work for each problem in the space provided. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed out work will not be graded.

- SHOW ALL YOUR WORK. You will be graded on the correctness and completeness of your methods as well as the accuracy of your final answers. Correct answers without supporting work may not receive full credit.

- Justifications require that you give mathematical (noncalculator) reasons and that you clearly identify functions, graphs, tables, or other objects you use.

- You are permitted to use your calculator in Part A to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate in your exam booklet the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

- Your work must be expressed in mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt($X^2$, $X$, 1, 5).

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

- Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

THE EXAM BEGINS ON THE NEXT PAGE

PLEASE TURN OVER

1. The position of a particle moving on the $x$-axis at time $t > 0$ seconds is: $x(t) = e^t - \sqrt{t}$ feet.
   (a) Find the average velocity of the particle over the interval $1 \le t \le 3$.
   (b) In what direction and how fast is the particle moving at $t = 1$ seconds?
   (c) When is the particle moving to the right?
   (d) Find the position of the particle when its velocity is zero.

a)

$x(t) = e^t - \sqrt{t}$

$v(t) = e^t - \frac{1}{2}t^{-1/2}$

$= e^t - \frac{1}{2\sqrt{t}}$

a)

$= \frac{1}{3-1}\int_1^3 \left(e^t - \frac{1}{2\sqrt{t}}\right) dt$

$= 8.3176$

b) $v(t) = e^t - \frac{1}{2\sqrt{t}} = 0$

$v(1) = e^1 - \frac{1}{2} = e - \frac{1}{2} = 2.218 > 0$ so right

c) $v(t) > 0$

$e^t - \frac{1}{2\sqrt{t}} > 0$

$t = 0.1756$

$v(t)$: + −

0 0.1756

2. Water flowed into a tank at an increasing rate $r(t)$ from $t = 0$ to $t = 5$ minutes. The rate of flow, $r(t)$, in cubic meters per minute ($m^3$/min), was measured at one minute intervals with the result shown in the table below.

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $r(t)$ | 4 | 5 | 7 | 11 | 12 | 14 |

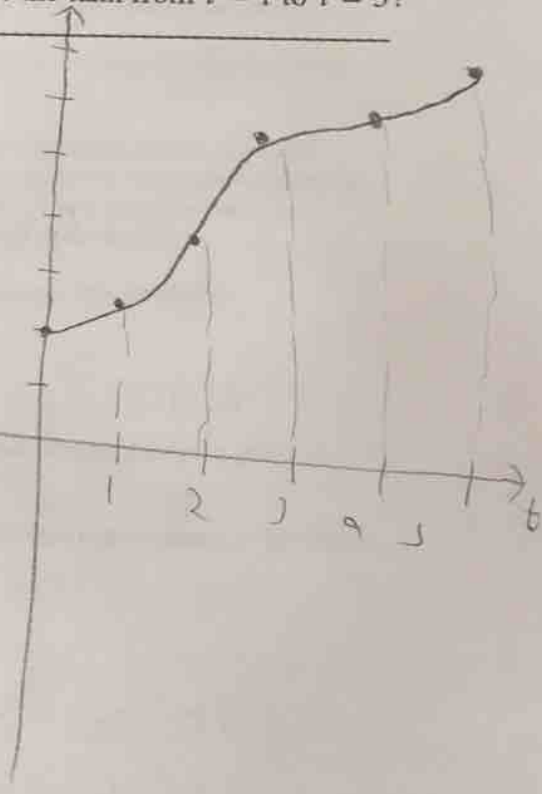
(a) Give the best upper and lower estimates for the total amount of water that flowed into the tank for $0 \le t \le 5$. Indicate units of measure.

(b) Suppose you use the average of the upper and lower estimates found in part (a) as your approximation for the total amount of water that flowed into the tank, what is the maximum error for this approximation?

(c) You are now informed that for $1 \le t \le 3$ the rate of flow was exactly $r(t) = t^2 - t + 5$ $m^3$/min. What is the exact amount of water that flowed into the tank from $t = 1$ to $t = 3$?

$\int_0^5 R(t)\,dt = [S(t)]_0^5$

$= S(5) - S(0)$



$\int_0^5 r(t)\,dt$

$=$

$L = \frac{5-0}{5}\left[R(0)+R(1)+R(2)+R(3)+R(4)\right]$

$= 4+5+7+11+12$

$= 39\ m^3$

$U = 5+7+11+12+14$

$\frac{1}{2}(39+49)$

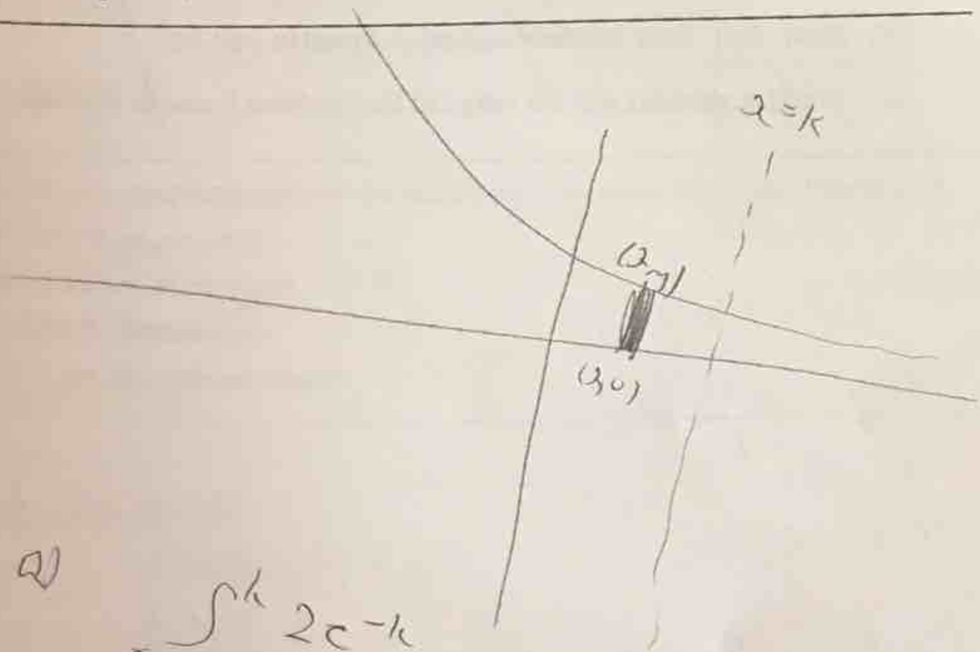
$\int_0^5 R(t) = 49$

3. Let $R$ be the first quadrant region enclosed by the graph of $y = 2e^{-x}$ and the line $x = k$.

(a) Find the area of $R$ in terms of $k$.

(b) Find the volume of the solid generated when $R$ is rotated about the $x$-axis in terms of $k$.

(c) What is the volume in part (b) as $k \to \infty$?




a)

$\int_0^k 2e^{-k}$

$= [-2e^{-x}]_0^k$

$= -2e^{-k} + 2e^0$

$= \frac{-2}{e^k} + 2$

b) $R(x) = y - 0 = (2e^{-x})^2$

Volume $= \pi \int_0^k (4e^{-2x})\,dx$

$= \pi [-2e^{-2x}]_0^k$

$= \pi(-2e^{-2k} + 2e^0)$

$= 2\pi\left(1 - \frac{1}{e^{2k}}\right)$

c) $\lim_{k \to \infty} \left(2\pi - \frac{2\pi}{e^k}\right) = 2\pi - \frac{2\pi}{\infty}$

$= 2\pi - 0$

$= 2\pi$

$\frac{2e^{-2x}}{-2}$

A CALCULATOR MAY **NOT** BE USED ON THIS PART OF THE EXAMINATION.
DURING THIS 45 MINUTES YOU MAY GO BACK AND WORK ON PART A.

4. Consider the differential equation $\frac{dy}{dx} = ye^{2x}$.

(a) Find a solution $y = f(x)$ to the differential equatiion satisfying $f(0) = 1$.

(b) Find $\lim\limits_{x \to -\infty} f(x)$ for the function $f$ found in part (a).

(c) Find the domain and the range of the function found in part (a).

$\frac{dy}{dx} = ye^{2x}$

a) $\int \frac{dy}{y} = \int e^{2x} dx$

$\ln y = \frac{e^{2x}}{2} + c$

$f(0) = 1$

$\ln 1 = \frac{e^0}{2} + c$

$0 = e^0 + 2c$

$c = \frac{-1}{2}$

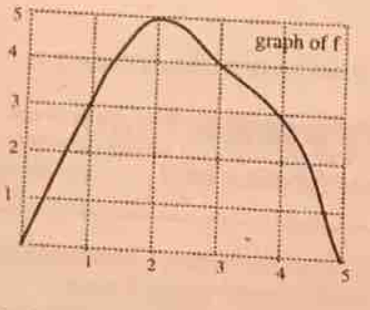
$\ln y = \frac{e^{2x}}{2} - \frac{1}{2}$

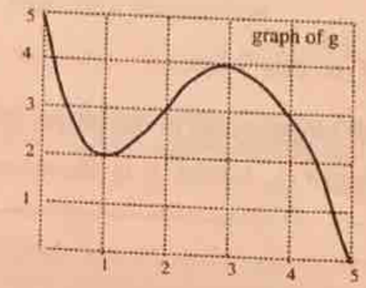
$2\ln y = e^{2x-1}$

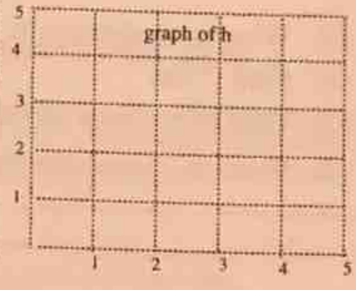
$y^2 = e^{e^{2x-1}}$

$y =$

5. Graphs of functions $f$ and $g$ are shown below.




Let $h(x) = f[g(x)]$. Use the graphs to answer the following questions about the function $h$.

(a) Approximate the critical points of $h$.

(b) Where does $h$ have a local minimum? $x = 3$

(c) Where is the function $h$ decreasing?

(d) Sketch a graph of $h$ on the axes provided.

a) $h'(x) = 0$

$f'[g(x)] \cdot g'(x) = 0$

$g'(x) = 0$ or $f'[g(x)] = 0$

$x = 1, 3$ $\quad g(x) = 2$

$x = 1,\ x = 4.5$

$x = 1, 3, 4.5$

c) $1 < x < 3$

$4.5 < x < 5$

b) $h'(x) = 0$

$h'(x)$: + − + −

0 1 3 4.5 5

$h'(x) = f'[g(x)] \cdot g'(x)$

$x = \frac{1}{2}$ ; $f'(3) \cdot (-) = (-)(-) = +ve$

$x = 2$ ; $f'(3) \cdot (+) = (-)(+) = -$

$x = 4$ ;

$x = 3$

$(3, 3)$ $\quad h(3) = f[g(3)] = f(4) = 3$

6. Let $f$ be the function defined by $f(x) = \ln\left(\dfrac{x}{x+1}\right)$.

(a) What is the domain of $f$?

(b) Find $f'(x)$.

(c) Find an equation for the tangent line to the graph of $f$ at the point $(1, f(1))$.

(d) Write an expression for $g'(x)$, where $g$ is the inverse function of $f$.

EXAM VI
CALCULUS AB
SECTION I, PART A
MULTIPLE-CHOICE
Time–55 minutes
Number of questions–28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:
The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

1. What is the $x$-coordinate of the point of inflection on the graph of $y = xe^x$?

(A) –2 (B) –1 (C) 0 (D) 1 (E) 2

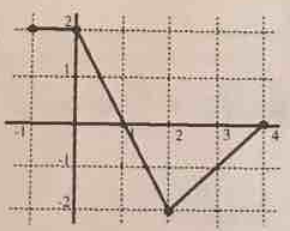
$y = xe^x$
$y' = e^x + xe^x$
$y'' = e^x + e^x + xe^x$
$2e^x + xe^x = 0$
$xe^x = -2e^x$
$x = -2$

Ans

2. The graph of a piecewise-linear function $f$, for $-1 \le x \le 4$, is shown in the figure. If the function $H$ is defined by

$$H(x) = \int_{-1}^{x} f(t)\,dt, \text{ for } -1 \le x \le 4, \text{ then } H(4) =$$

Graph of $f$




$H(4) = \int_{-1}^{4} f(t) = \frac{1}{2}(3)(2) + \frac{1}{2}(1)(2) + 1(2)$
$= -3 + 1 + 2$
$= 0$

(A) –2 (B) –1 (C) 0 (D) 1 (E) 2

$H(4) = \int_{-1}^{4} f(t)\,dt$
$= 1(2) + \frac{1}{2}(1)(2) - \frac{1}{2}(3)(2)$

Ans

3. $\int_0^2 |x-1|\,dx =$

$x \geq 1,\ x-1$
$x < 1,\ -x+1$

(A) 0
(B) 1
(C) $\frac{1}{2}$
(D) 2
(E) 3

$\int_0^1 (-x+1)\,dx + \int_1^2 (x-1)\,dx$
$= \left[-\frac{x^2}{2}+x\right]_0^1 + \left[\frac{x^2}{2}-x\right]_1^2$
$= -\frac{1}{2}+1+\frac{4}{2}-2-\left(\frac{1}{2}-1\right)$
$= -\frac{1}{2}+1-\frac{1}{2}+1$
$= 1$

Ans

4. The function $f$ is continuous at the point $(c, f(c))$. Which of the following statements could be false?

(A) $\lim_{x\to c} f(x)$ exists
(B) $\lim_{x\to c} f(x) = f(c)$
(C) $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)$
(D) $f(c)$ is defined
(E) $f'(c)$ exists

Ans

5. $\int_0^x 2\sec^2\left(2t+\frac{\pi}{4}\right) dt =$

$2\left[\frac{\tan(2t+\pi/4)}{2}\right]$
$= \left[\tan(2t+\pi/4)\right]_0^x$
$\tan(2x+\pi/4)$
$-\tan(\pi/4)$
$= \tan(2x+\pi/4)-1$

(A) $2\tan\left(2x+\frac{\pi}{4}\right)$
(B) $2\tan\left(2x+\frac{\pi}{4}\right)-2$
(C) $\tan\left(2x+\frac{\pi}{4}\right)-1$
(D) $2\sec\left(2x+\frac{\pi}{4}\right)\tan\left(2x+\frac{\pi}{4}\right)$
(E) $\sec\left(2x+\frac{\pi}{4}\right)\tan\left(2x+\frac{\pi}{4}\right)$

Ans

6. If $xy + x^2 = 6$, then the value of $\frac{dy}{dx}$ at $x = -1$ is

(A) $-7$ (B) $-2$ (C) 0 (D) 1 (E) 3

Ans

7. $\int_2^3 \frac{x}{x^2+1}\,dx =$

(A) $\frac{1}{2}\ln\frac{3}{2}$ (B) $\frac{1}{2}\ln\frac{1}{2}$ (C) $\frac{1}{2}\ln 2$ (D) $2\ln 2$ (E) $\frac{1}{2}\ln 5$

$\left[\frac{1}{2}\ln(x^2+1)\right]_2^3$

$= \frac{1}{2}\ln 10 - \frac{1}{2}\ln 5$

$= \frac{1}{2}\ln(2)$

Ans

8. Suppose that $g$ is a function with the following two properties:

(i) $g(-x) = g(x)$ for all $x$; and Even

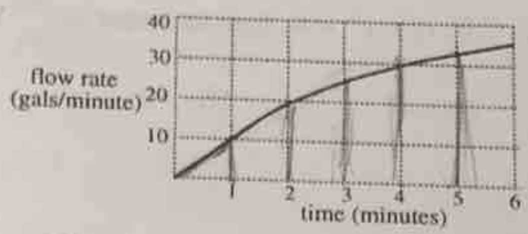
(ii) $g'(a)$ exists.

Which of the following must necessarily be equal to $g'(-a)$?

(A) $g'(a)$ (B) $-g'(a)$ (C) $\frac{1}{g'(a)}$ (D) $-\frac{1}{g'(a)}$ (E) none of these

Ans

9. Water flows into a tank at a rate shown in the figure. Of the following, which best approximates the total number of gallons in the tank after 6 minutes?




(A) 85 (B) 95 (C) 105 (D) 125 (E) 135

$\frac{1}{2}(0+10)+\frac{1}{2}(10+20)+\frac{1}{2}(20+25)+\frac{1}{2}(25+30)+\frac{1}{2}(30+32)+\frac{1}{2}(32+35)$

Ans

---

10. What is the instantaneous rate of change at $x = 0$ of the function $f$ given by $f(x) = e^{2x} - 3\sin x$?

(A) $-2$ (B) $-1$ (C) 0 (D) 4 (E) 5

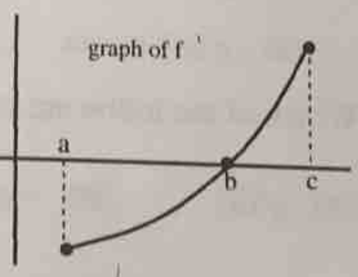
$f'(x) = 2e^{2x} - 3\cos x$
$f'(0) = 2e^0 - 3\cos 0$
$= 2 - 3$
$= -1$

Ans

---

11. Suppose $f$ is a function with continuous first and second derivatives on the closed interval $[a, c]$. If the graph of its derivative $f'$ is given in the figure, which of the following is true?

(A) $f$ is increasing on the interval $(a, b)$ No
(B) $f$ has a relative maximum at $x = b$. No
(C) $f$ has an inflection point at $x = b$. No
(D) The graph of $f$ is concave down on the interval $(a, b)$. No
(E) $\int_a^c f'(x)\,dx = f(c) - f(a)$




Ans

12. Suppose $F(x) = \int_0^{x^2} \frac{1}{2+t^3}\,dt$ for all real $x$, then $F'(-1) =$

(A) 2 (B) 1 (C) $\frac{1}{3}$ (D) $-2$ (E) $-\frac{2}{3}$

$F'(x) = \frac{1}{2+x^6}(2x)$

$F'(-1) = \frac{1}{2+1}(-2)$

$= \frac{-2}{3}$

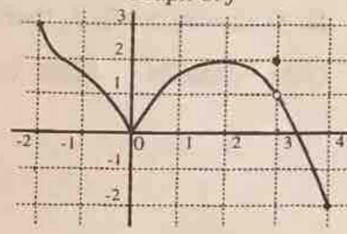
Ans

13. The graph of the function $f$ is shown in the figure. For what values of $x$, $-2 < x < 4$, is $f$ not differentiable?

(A) 0 only
(B) 0 and 2 only
(C) 2 and 3 only
(D) 0 and 3 only
(E) 0, 1 and 3 only

$x = 0, x = 3$

Graph of $f$




? is it continuous at $x=3$

Ans

14. A particle moves along the $x$-axis so that its position at any time $t \geq 0$ is given by $x(t) = \frac{t}{t^2+4}$. The particle is at rest when $t =$

(A) 0 (B) $\frac{1}{4}$ (C) 1 (D) 2 (E) 4

$x(t) = \frac{t}{t^2+4}$

$v(t) = \frac{(t^2+4)(1) - t(2t)}{(t^2+4)^2}$

$\frac{t^2+4-2t^2}{(t^2+4)^2}$

$\frac{4-t^2}{(t^2+4)^2} = 0$

$4-t^2=0$
$t=2$

$t \geq 0$

Ans

15. Find the maximum value of $f(x)=2x^3+3x^2-12x+4$ on the closed interval [0,2].

(A) $-3$

(B) 0

(C) 2

(D) 4

(E) 8

ANS 8

$f'(x)=6x^2+6x-12=0$
$x^2+x-2=0$
$(x-1)(x+2)=0$
$x=1, x=-2$

$f(x)$: 0 $-$ 1 $+$ 2

$f(0)=4$
$f(2)=8$

Ans

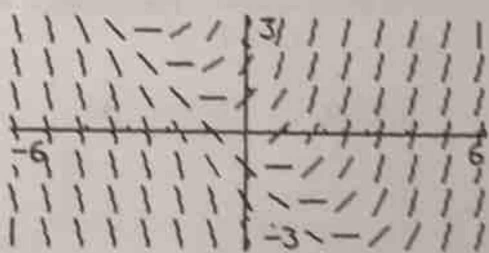
16. If $f(x)=\ln(\cos 2x)$, then $f'(x)=$

(A) $-2\tan 2x$ (B) $\cot 2x$ (C) $\tan 2x$ (D) $-2\cot 2x$ (E) $2\tan 2x$

$f'(x)=\frac{-2\sin 2x}{\cos 2x}$
$=-2\tan 2x$

Ans

17. The slope field for a differential equation $\frac{dy}{dx}=f(x,y)$ is given in the figure. The slope field corresponds to which of the following differential equations?




(A) $\frac{dy}{dx}=x+y$

(B) $\frac{dy}{dx}=-y$

(C) $\frac{dy}{dx}=y-\frac{1}{2}y^2$

(D) $\frac{dy}{dx}=x^2+y^2$

(E) $\frac{dy}{dx}=y^2$

Ans

18. The $y$-intercept of the tangent line to the curve $y = \sqrt{x+3}$ at the point (1, 2) is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) $\frac{5}{4}$ (E) $\frac{7}{4}$

at (1,2) $y' = \frac{1}{2}(x+3)^{-1/2}$

$y' = \frac{1}{2}(4)^{-1/2} = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

$y - 2 = \frac{1}{4}(x-1)$

$4y - 8 = x - 1$

$4y = x + 7$

$x = 0$

$4y = 7$

$y = \frac{7}{4}$

Ans

19. The function defined by $f(x) = (x-1)(x+2)^2$ has inflection points at $x =$

(A) –2 only

(B) –1 only

(C) 0 only

(D) –2 and 0 only

(E) –2 and 1 only

Ans

20. If $\int_0^b (4bx - 2x^2)\,dx = 36$, then $b =$

(A) –6

(B) –3

(C) 3

(D) 6

(E) 15

$\left[2bx^2 - \frac{2}{3}x^3\right]_0^b = 36$

$(2b^3 - \frac{2}{3}b^3) - (0) = 36$

$\frac{4}{3}b^3 = 36$

$b^3 = 36 \cdot \frac{3}{4}$

$b^3 = 27$

$b = 3$

Ans

21. If $\frac{dy}{dx} = -10y$ and if $y = 50$ when $x = 0$, then $y =$

(A) $50\cos 10x$

(B) $50e^{10x}$

(C) $50e^{-10x}$

(D) $50 - 10x$

(E) $50 - 5x^2$

$\frac{dy}{dx} = -10y$ $\quad$ $y = 50$ when $x = 0$

$dy = -10y\,dx$

$\frac{dy}{y} = -10\,dx$

$\ln y = -10x + c$

$\ln 50 = 0 + c$ $\quad$ $y = 50e^{-10x}$

$\ln y = -10x + \ln 50$

$y = e^{-10x + \ln 50}$

Ans

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22. If $f(x) = x^3 - 5x^2 + 3x$, then the graph of $f$ is decreasing and concave down on the interval

(A) $\left(0, \frac{1}{3}\right)$ (B) $\left(\frac{1}{3}, \frac{2}{3}\right)$ (C) $\left(\frac{1}{3}, \frac{5}{3}\right)$ (D) $\left(\frac{5}{3}, 3\right)$ (E) $(3, \infty)$

$f'(x) = 3x^2 - 10x + 3 = 0$ $\quad$ $f''(x) = 6x - 10 = 0$

$3x^2 - x - 9x + 3 = 0$ $\quad$ $6x = 10$

$x(3x-1) - 3(3x-1) = 0$ $\quad$ $x = \frac{5}{3}$

$(x-3)(3x-1) = 0$

$x = 3,\ x = \frac{1}{3}$

$f'(x)$: $+$ $-$ $+$ ; $\frac{1}{3}$, $3$

dec on $(\frac{1}{3}, 3)$

$-$ $+$ ; $\frac{5}{3}$

$(-\infty, \frac{5}{3})$

Ans

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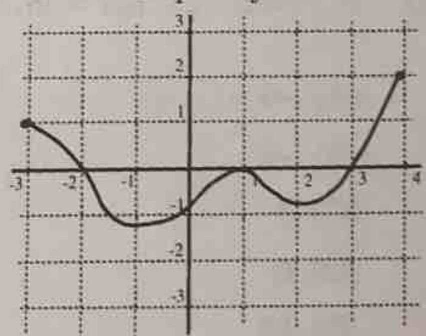
23. The figure shows the graph of $f'$, the derivative of a function $f$. The domain of $f$ is the closed interval $[-3,4]$. Which of the following is true?

Graph of $f'$



I. $f$ is increasing on the interval $(2,4)$. No

II. $f$ has a relative minimum at $x = -2$. No

III. The $f$-graph has an inflection point at $x = 1$. Yes

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, III

Ans

24. How many critical values does the function $f(x) = \arctan(2x - x^2)$ have?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

$2x-2x$

$f(x) = \tan^{-1}(2x-x^2)$

$f'(x) = \dfrac{2-2x}{1+(2x-x^2)^2}$

$2-2x=0$

$2x=2$

$x=1$

$1+(2x-x^2)^2 \neq 0$

Ans

25. Which of the following is continuous at $x = 1$?

I. $f(x) = |x-1|$
II. $f(x) = e^{x-1}$
III. $f(x) = \ln(e^{x-1} - 1)$

(A) I only
(B) II only
(C) I and II only
(D) II and III only
(E) I, II, III

$\lim_{x\to1^+} x-1=0$ $\lim_{x\to1^-} 1-x=0$

$\lim_{x\to1^+} e^{x-1}=e^0$ $\lim_{x\to1^-} e^{x-1}=e^0$

$e^{x-1}-1>0$

$\lim_{x\to1^+} e^{x-1}-1=1-1=0$

$\lim_{x\to1^-} e^{x-1}-1=$ doesnt exist

Domain: $e^{x-1}-1>0$

$e^{x-1}>1$

$x-1>\ln 1$

$x>1$

$x-1$

$x>1,\ x\geq 1$

$x<1$

Ans

26. The number of motels per mile along a 5 mile stretch of highway approaching a city is modeled by the function $m(x) = 11 - e^{0.2x}$, where $x$ is the distance from the city in miles. The approximate number of motels along that stretch of highway is

(A) 16
(B) 26
(C) 36
(D) 46
(E) 56

$m(x) = 11 - e^{0.2x}$

$S(x) = \int_0^5 11 - e^{0.2x}$

$= \left[11x - \dfrac{e^{0.2x}}{0.2}\right]_0^5$

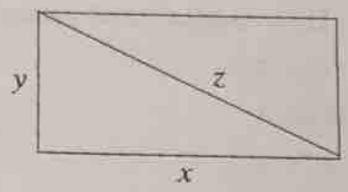
$= 55 - \dfrac{e^1}{0.2} - \left(-\dfrac{e^0}{0.2}\right)$

$= 55 - 5e + 5$

$= 60 - 5e$

Ans

27. The diagonal $z$ of the rectangle at the right is increasing at the rate of 1 cm/sec and $\frac{dy}{dt} = 3\frac{dx}{dt}$. At what rate is the length $x$ increasing when $x = 3$ cm and $y = 4$ cm ?




(A) 1 cm/sec

(B) $\frac{1}{2}$ cm/sec

(C) $\frac{1}{3}$ cm/sec

(D) $\frac{1}{4}$ cm/sec

(E) $\frac{1}{15}$ cm/sec

$\frac{dz}{dt} = 1$

$\frac{dy}{dt} = 3\frac{dx}{dt}$

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$2(3)\frac{dx}{dt} + 2(4)(3)\frac{dx}{dt} = 2(5)(1)$

$6\frac{dx}{dt} + 24\frac{dx}{dt} = 10$

$30\frac{dx}{dt} = 10$

$\frac{dx}{dt} = \frac{10}{30} = \frac{1}{3}$

$3^2 + 4^2 = z^2$
$z^2 = 25$
$z = 5$

Ans

28. If $f(x) = \sin(2x) + \ln(x+1)$, then $f'(0) =$

(A) $-1$ (B) 0 (C) 1 (D) 2 (E) 3

$f'(x) = 2\cos 2x + \frac{1}{x+1}$

$f'(0) = 2\cos 0 + 1$

$= 2(1) + 1$

$= 3$

Ans

EXAM VI
CALCULUS AB
SECTION I PART B
MULTIPLE-CHOICE
CALCULATORS
Time–50 minutes
Number of questions–17

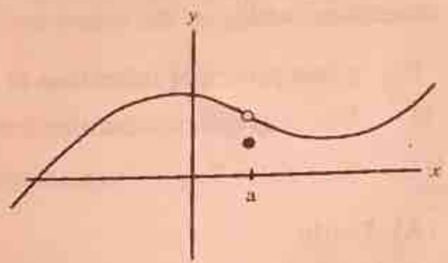
Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS IN THIS PART OF THE EXAMINATION.

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

1. The graph of a function $f$ is shown to the right. Which of the following statements about $f$ is false?




(A) $f$ has a relative minimum at $x = a$. F

(B) $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$ T

(C) $\lim_{x\to a} f(x) \neq f(a)$ T

(D) $f(a) > 0$ T

(E) $f'(a) < 0$ ? T

Ans

2. The function $f$ defined by $f(x) = e^{3x} + 6x^2 + 1$ has a horizontal tangent at $x =$

(A) $-0.144$ (B) $-0.150$ (C) $-0.156$ (D) $-0.162$ (E) $-0.168$

$f(x) = e^{3x} + 6x^2 + 1$
$f'(x) = 3e^{3x} + 12x = 0$

Ans

3. Boyle's Law states that if the temperature of a gas remains constant, then the pressure $P$ and the volume $V$ of the gas satisfy the equation $PV = c$ where $c$ is a constant. If the volume is decreasing at the rate of 10 $in^3$ per second, how fast is the pressure increasing when the pressure is 100 $lb/in^2$ and the volume is 20 $in^3$?

(A) 5 $lb/in^2$ (B) 10 $lb/in^2$ (C) 50 $lb/in^2$ (D) 200 $lb/in^2$ (E) 500 $lb/in^2$

$PV = c$

$\frac{dP}{dt}V + P\frac{dV}{dt} = 0$

$\frac{dP}{dt}(20) + 100(-10) = 0$

$\frac{dP}{dt} = \frac{1000}{20} = 50$

$\frac{dV}{dt} = -10$

we want $\frac{dP}{dt}$ when $P = 100$, $V = 20$

Ans

4. The graph of the second derivative of a function $g$ is shown in the figure. Use the graph to determine which of the following are true.

I. $g$ has points of inflection at $x = 1$ and $x = 3$. F

II. The $g$-graph is concave down on the interval (3, 4). T

III. If $g'(0) = 0$, $g$ is increasing at $x = 2$. T ???

(A) I only

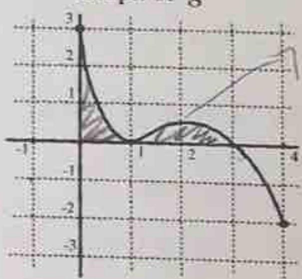
(B) II only

(C) II and III only

(D) I and II only

(E) I, II, III

Graph of $g''$



why max ???

Ans

5. A particle moves along a straight line with its position at any time $t \geq 0$ given by

$s(t) = \int_0^t (x^3 - 2x^2 + x)\, dx$, where $s$ is measured in meters and $t$ in seconds. The maximum velocity attained by the particle on the interval $0 \leq t \leq 3$ is

(A) 0.333 m/sec

(B) 0.148 m/sec

(C) 1 m/sec

(D) 3 m/sec

(E) 12 m/sec

$V(t) = t^3 - 2t^2 + t$

$a(t) = 3t^2 - 4t + 1 = 0$

$3t^2 - 3t - t + 1 = 0$

$3t(t-1) - (t-1) = 0$

$(3t-1)(t-1)$

$t = \frac{1}{3}, t = 1$

$a(t)$: + − + (0, 1/3, 1, 3)

$V(0) = 0$

$V(1/3) = (1/3)^3 - 2(1/3)^2 + 1/3$

$= \frac{1}{27} - \frac{2}{9} + \frac{1}{3} = 0.\overline{148}$

$V(3) = 12$

Ans

6. If $\frac{dy}{dx} = \sqrt{2x+1}$, then the average rate of change $y$ with respect to $x$ on the closed interval $[0, 4]$ is

(A) 13 (B) $\frac{9}{2}$ (C) $\frac{13}{2}$ (D) $\frac{13}{6}$ (E) $\frac{1}{9}$

$\frac{1}{4-0} \int_0^4 \sqrt{2x+1}\, dx$

$= \frac{13}{6}$

Ans

7. If $f'$ is a continuous function on the closed interval $[0, 2]$ and $f(0) = f(2)$, then $\int_0^2 f'(x)\, dx =$

$F(2) - F(0) = 0$

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

Ans

8. If $k \neq 0$, then $\lim_{x \to k} \frac{x^2 - k^2}{x^2 - kx} =$

(A) 0
(B) 2
(C) $2k$
(D) $4k$
(E) nonexistent

$\lim_{x \to k} \frac{(x+k)(x-k)}{x(x-k)} = \frac{x+k}{x}$

$\lim_{x \to k} \left(\frac{x+k}{x}\right) = \frac{2k}{k} = 2$

Ans

9. Suppose that, during the first year after its hatching, the weight of a duck increases at a rate proportional to its weight. The duckling weighed 2 pounds when it was hatched and 3.5 pounds at age 4 months. How many pounds will the bird weigh at age 6 months?

(A) 4.2 lbs
(B) 4.6 lbs
(C) 4.8 lbs
(D) 5.6 lbs
(E) 6.5 lbs

$\frac{dW}{dt} = kW$

$\int \frac{dW}{W} = k\,dt$

$\ln W = kt + C$

$W = Ce^{kt}$

$2 = Ce^{kt}$ $W = 2,\ t = 0$

$2 = Ce^0$

$C = 2$

$W = 2e^{kt}$

$3.5 = 2e^{4k}$

$e^{4k} = 1.75$

$4k = \ln 1.75$

$k = 0.1399$

$W =$

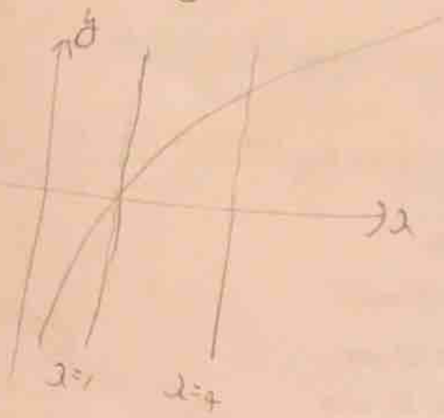
$W = 2e^{6k}$
$= 4.630$
$= 4.6$

Ans

10. Let $R$ be the region in the first quadrant enclosed by the $x$-axis and the graph of $y = \ln x$ from $x = 1$ to $x = 4$. If the Trapezoid Rule with 3 subdivisions is used to approximate the area of $R$, the approximation is

ANS

(A) 1.242 (B) 2.485 (C) 4.970 (D) 7.078 (E) 14.156




Area $= \int_1^4 \ln x \, dx$

Area $= \frac{4-1}{2(3)}\left[f(1)+2(f(2)+f(3))+f(4)\right]$

$= \frac{1}{2}(\ln 1 + 2(\ln 2 + \ln 3) + \ln 4) = 2.485$

Ans

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11. A solid has as its base the region enclosed by the graph of $y = \cos x$ and the $x$-axis between $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$. If every cross section perpendicular to the $x$-axis is a square, the volume of the solid is

(A) $\frac{\pi}{4}$

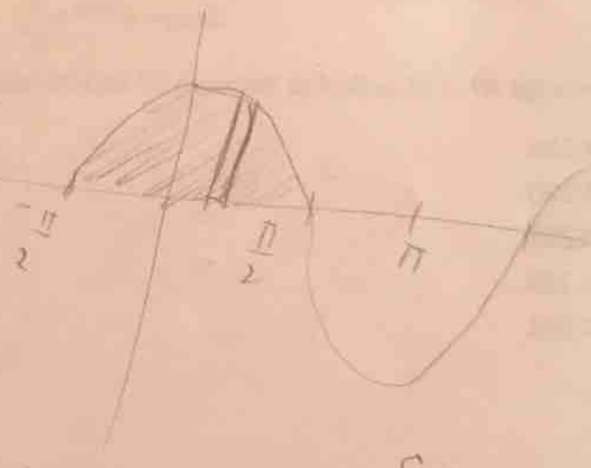
(B) $\frac{\pi^2}{4}$

(C) $\frac{\pi}{2}$

(D) $\frac{\pi^2}{2}$

(E) 2

$R(x) = 4 - 0$
$= \cos x$


Work $= \int_{-\pi/2}^{\pi/2} (\cos^2 x)\,dx$

$\cos 2x = 2\cos^2 x - 1$

$\cos^2 x = \frac{1}{2} + \frac{\cos 2x}{2}$

$= \left[\frac{1}{2}x + \frac{\sin 2x}{4}\right]_{-\pi/2}^{\pi/2}$

$= \frac{1}{2}\left(\frac{\pi}{2}\right) + 0 - \left(\frac{-\pi}{4} + 0\right)$

Ans



$= \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

12. If the function $f$ is differentiable at the point $(a, f(a))$, then which of the following are true?

I. $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

II. $f'(a) = \lim_{h \to 0} \frac{f(a) - f(a-h)}{h}$

III. $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a-h)}{2h}$

(A) I only

(B) I and II only

(C) I and III only

(D) II and III only

(E) I, II, III

Ans

13. The level of air pollution at a distance $x$ miles from a tire factory is given by

$$L(x) = e^{-0.1x} + \frac{1}{x^2}.$$

The average level of pollution between 10 and 20 miles from the factory is

(A) 0.226

(B) 0.230

(C) 0.234

(D) 0.238

(E) 0.242

$\frac{1}{20-10} \int_{10}^{20} (e^{-0.1x} + \frac{1}{x^2})\,dx$

$\approx 0.2375$

Ans

14. Suppose the continuous function $f$ is defined on the closed interval $[0, 3]$ such that its derivative $f'$ is defined by $f'(x) = e^x \sin(x^2) - 1$. Which of the following are true about the graph of $f$?

    I. $f$ has exactly one relative maximum point. T
    II. $f$ has two relative minimum points. T
    III. $f$ has two inflection points. F

    no 3

    (A) I only
    (B) II only
    (C) III only
    (D) I and II only
    (E) I, II, III

    $f'(x) = e^x \sin(x^2) - 1$
    $f''(x) = e^x \sin(x^2) + e^x(2x)\cos(x^2)$
    $= e^x \sin(x^2) + 2xe^x \cos(x^2)$

Ans

15. If the average value of $y = x^2$ over the interval $[1, b]$ is $\frac{13}{3}$, then the value of $b$ could be

    (A) $\frac{7}{3}$ (B) 3 (C) $\frac{11}{3}$ (D) 4 (E) $\frac{13}{3}$

    $\frac{1}{b-1}\int_1^b x^2\,dx = \frac{13}{3}$
    $\frac{1}{b-1}\left[\frac{x^3}{3}\right]_1^b = \frac{13}{3}$
    $\frac{1}{b-1}\left(\frac{b^3}{3} - \frac{1}{3}\right) = \frac{13}{3}$
    $\frac{1}{b-1}\left(\frac{b^3-1}{3}\right) = \frac{13}{3}$
    $\frac{b^3-1}{3b-3} = \frac{13}{3}$
    $3b^3 - 3 = 39b - 39$
    $3b^3 - 39b + 36 = 0$
    $b^3 - 13b + 12 = 0$
    $b = -4, 3, 1$

Ans

16. If the function $f$ is defined on the closed interval [0, 3] by $f(x) = \dfrac{2x}{x^2+1}$, which of the following is true?

I. $\int_0^3 f(x)\,dx = \ln 10$ T

II. $f$ has a relative maximum at $x = 1$.

III. $f'(2) = \frac{1}{2}$

$f'(x) = \dfrac{(x^2+1)(2) - 2x(2x)}{(x^2+1)^2}$

$= \dfrac{2x^2+2-4x^2}{(x^2+1)^2}$

$= \dfrac{-2x^2+2}{(x^2+1)^2}$

$f'(2) = \dfrac{-2(4)+2}{5^2} = -\dfrac{6}{25}$

(A) I only
(B) II only
(C) I and II only
(D) II and III only
(E) I, II, III

Ans

17. The area of the region bounded by the graphs of $y = \arctan x$ and $y = 4 - x^2$ is approximately

(A) 10.80
(B) 10.97
(C) 11.14
(D) 11.31
(E) 11.48

Ans

EXAM VI
CALCULUS AB
SECTION II, PART A
Time–45 minutes
Number of questions–3

A GRAPHING CALCULATOR IS REQUIRED ON THIS PART OF THE EXAMINATION.

- Before you begin Part A of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems and you will be able to come back to Part A (without a calculator), if you have time after Part B. All problems are given equal weight, but the parts of a particular solution are not necessarily given equal weight.

- You should write all work for each problem in the space provided. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed out work will not be graded.

- SHOW ALL YOUR WORK. You will be graded on the correctness and completeness of your methods as well as the accuracy of your final answers. Correct answers without supporting work may not receive full credit.

- Justifications require that you give mathematical (non calculator) reasons and that you clearly identify functions, graphs, tables, or other objects you use.

- You are permitted to use your calculator in Part A to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate in your exam booklet the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

- Your work must be expressed in mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt($X^2$, $X$, 1, 5).

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

- Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

THE EXAM BEGINS ON THE NEXT PAGE

PLEASE TURN OVER

1. Let $R$ be the region in the first quadrant under the graph of $y = \dfrac{8}{\sqrt[3]{x}}$ for $1 \le x \le 8$.

   (a) Find the area of $R$.

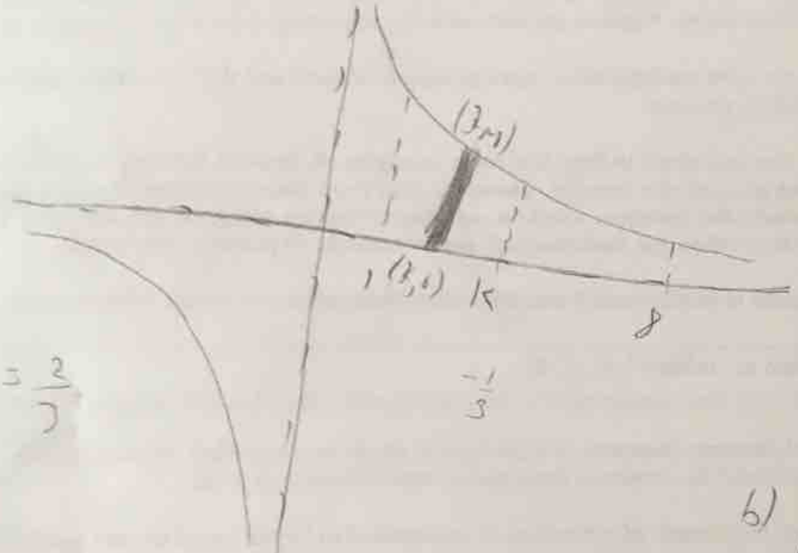
   (b) The line $x = k$ divides the region $R$ into two regions. If the part of region $R$ to the left of the line is $\frac{5}{12}$ of the area of the whole region $R$, what is the value of $k$?

   (c) Find the volume of the solid whose base is the region $R$ and whose cross sections cut by planes perpendicular to the $x$-axis are squares.

192

$\lim_{x \to \infty} y = 0$ $\quad y = 8x^{-1/3}$ $\quad x \neq 0$ $\quad \frac{8}{x^{1/3}} \neq 0$ $\quad \frac{1}{x}$

a)




$\frac{2}{3} + \frac{3}{3} = \frac{5}{3}$

$-\frac{1}{3} + \frac{3}{3} = \frac{2}{3}$ $\quad -\frac{1}{3}$

$= \frac{3}{8}$

b) $8x^{-1/3}$ $\quad 8x^{-1/3}$

$\int_1^8 8x^{-1/3}\,dx = \left[\frac{8x^{2/3}}{2/3}\right]_1^8$

$= \left[12x^{2/3}\right]_1^8$

$= 12\sqrt[3]{64} - 12$

$= 12(4) - 12$

$= 36$

$\left[192x^{1/3}\right]_1^8$

$= 192(2) - 192(1)$

$= 192$

Volume $= \int_1^8 (8x^{-1/3})(8x^{-1/3})\,dx$

$= \int_1^8 64x^{-2/3}\,dx = \left[\frac{64x^{1/3}}{1/3}\right]_1^8$

b) $\int_1^k 8x^{-1/3}\,dx = \frac{5}{12}(36)$

$= \left[12x^{2/3}\right]_1^k = 15$

$12k^{2/3} - 12 = 15$

$12k^{2/3} = 27$

$k^{2/3} = \frac{9}{4}$

$k = \left(\frac{9}{4}\right)^{3/2}$

$= 3.375$

2. A particle starts at the point $(1, 0)$ at $t = 0$ and moves along the $x$-axis so that at time $t \geq 0$ its velocity $v(t)$ is given by $v(t) = 1 + \frac{t}{1+t^2}$.

   (a) Determine the maximum velocity of the particle. Show your work.

   (b) Find an expression for the position $s(t)$ of the particle at time $t$.

   (c) What is the limiting value of the velocity as $t$ increases without bound?

   (d) Determine for which values of $t$, if any, the particle reaches the point $(101, 0)$.

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$V(t) = 1 + \frac{t}{1+t^2}$

a) $V'(t) = \frac{(1+t^2)(1) - t(2t)}{(1+t^2)^2} = \frac{1+t^2-2t^2}{(1+t^2)^2} = \frac{1-t^2}{(1+t^2)^2} = 0$

$1-t^2 = 0$, $t^2 = 1$, $t = \pm 1$ $\quad$ $1+t^2 = 0$, $t^2 \neq -1$

$V(0) = 1 + \frac{0}{1} = 1$

$V(1) = 1 + \frac{1}{2} = 1.5$

$V(t)$: $-$ $+$ $-$; $0$, $-1$, $1$

b) $S(t) = \int \left(1 + \frac{t}{1+t^2}\right) dx$

$= t + \frac{1}{2}\ln|1+t^2| + C$

$(1, 0)$

$0 = 1 + \frac{1}{2}\ln 2 + C$

$-1 = \frac{1}{2}\ln 2 + C$

$-2 = \ln 2 + C$

$C = -2 - \ln 2$

$S(t) = t + \frac{1}{2}\ln|1+t^2| - 2 - \ln 2$

c) $\lim_{t\to\infty} 1 + \frac{t}{1+t^2} = \lim_{t\to\infty}\left(1 + \frac{\frac{1}{t}}{\frac{1}{t^2}+1}\right)$

$= 1 + \frac{0}{1}$

$= 1 + 0 = 1$

d)

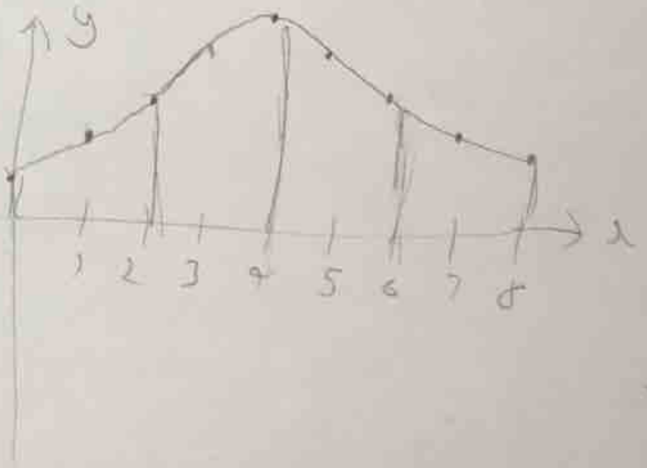
3. The rate at which an air-conditioning unit for a theater complex pumps out cool air, in metric tons per hour, is given by a differentiable function $R$ of time $t$. The table shows the rate as measured every hour over an 8-hour time period.

| $t$ (hours) | $R(t)$ (metric tons per hour) |
|---|---|
| 0 | 4.6 |
| 1 | 5.4 |
| 2 | 6.1 |
| 3 | 6.5 |
| 4 | 6.8 |
| 5 | 6.3 |
| 6 | 6.0 |
| 7 | 5.5 |
| 8 | 4.8 |

(a) Use a midpoint Riemann sum with 4 subintervals of equal length to approximate $\int_0^8 R(t)\,dt$. Explain, using correct units, the meaning of your answer in terms of air flow.

(b) Is there some time $t$, $0 < t < 8$, such that $R'(t) = 0$ ? Explain.

(c) The rate of air flow $R(t)$ can be approximated using $Q(t) = \frac{1}{8}\left(36 + 8t - t^2\right)$. Use $Q(t)$ to approximate the average rate of air flow during the 8-hour time period.



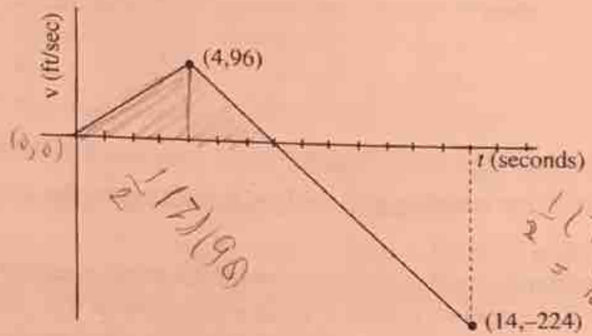

$= \frac{8-0}{4}\left[f(1) + f(3) + f(5) + f(7)\right]$

$= 2(5.4 + 6.5 + 6.3 + 5.5)$

$= 47.4$

A CALCULATOR MAY **NOT** BE USED ON THIS PART OF THE EXAMINATION.
DURING THIS 45 MINUTES YOU MAY GO BACK AND WORK ON PART A.

4. The graph below shows the velocity $v$ in feet per second of a small rocket that was fired from the top of a tower at time $t = 0$ ( $t$ in seconds). The rocket accelerated with constant upward acceleration until its fuel was expended, then fell back to the ground at the foot of the tower. The entire flight lasted 14 seconds.




Slope $= \frac{96}{4}$
$= 24$ ft/sec
$V(t) > 0$

$\frac{1}{2}(7)(224)$
$= 784$

(a) What was the acceleration of the rocket while its fueled lasted?

(b) How long was the rocket rising?

(c) What was the maximum height above the ground that the rocket reached?

(d) How high was the tower from which the rocket was fired? 448

a) $a = v'(t) = \frac{96-0}{4-0} = \frac{96}{4} = 24$ ft/sec$^2$

b) $v(t) > 0 \quad 0 < t < 7$

c) $v(t) = 0$

$t = 7$ - max

$S(7) = \int_0^7 v(t) = \frac{1}{2}(7)(96)$

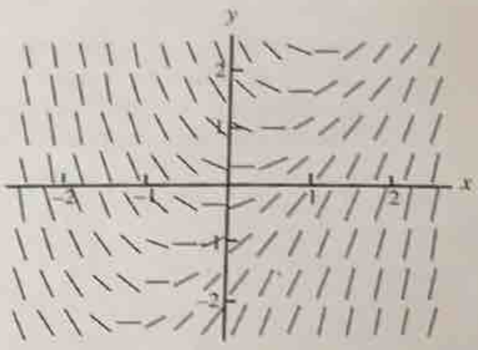
$= 336$ ft

$(7,0) \quad (336, 4)$

d) $784 - 336 = 448$

5. Consider the differential equation $\frac{dy}{dx} = 2x - y$.

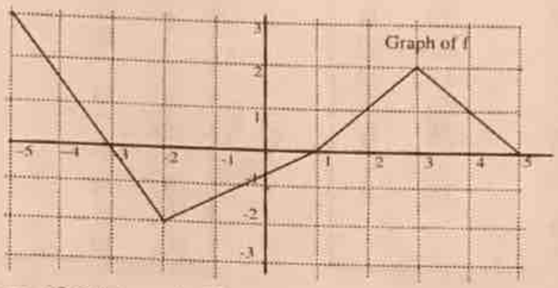
(a) The slope field for the given differential equation is provided. Sketch the solution curve that passes through the point (1, 0) and sketch the solution curve that passes through the point (0, 1).



(b) Find the value of $b$ for which $y = 2x + b$ is a solution to the differential equation. Justify your answer.

(c) Let $g$ be the function that satisfies the given differential equation with the initial condition $g(0) = 0$. Does the graph of $g$ have a local extremum at the point (0, 0)? If so, is the point a local maximum or a local minimum? Justify your answer.

6. Let $G(x) = \int_{-3}^{x} f(t)\,dt$ and $H(x) = \int_{2}^{x} f(t)\,dt$ where $f$ is the function graphed below.



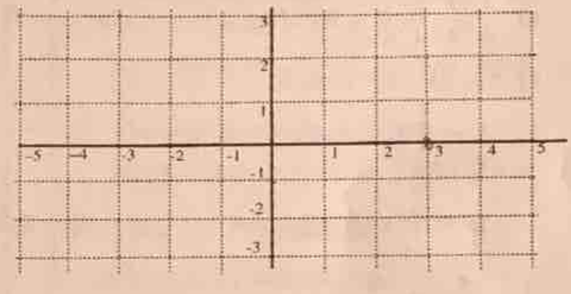

(a) How are the values of $G(x)$ and $H(x)$ related? Give a geometric explanation of this relationship.

(b) On which intervals of $[-5, 5]$, if any, is $H$ increasing?

(c) At what $x$-coordinates, $-5 < x < 5$, does $G$ have a relative maximum? Justify your answer.

(d) On which subintervals of $[-5, 5]$, if any, is $G$ concave up? $[-3,1)$ $(1,3)$

(e) Sketch a graph of $G$ on the axes provided.



$H'(2) > 0$

$f(2) > 0$

$G'(2) = 0$

$f(2) = 0$

$G'(2) > 0$

$f'(2) > 0$

$f(2)$ is increasing

# Answers

EXAM I SECTION I PART A

| | | | | | |
|---|---|---|---|---|---|
| 1. | A | 11. | A | 21. | C |
| 2. | B | 12. | E | 22. | B |
| 3. | E | 13. | C | 23. | D |
| 4. | C | 14. | B | 24. | D |
| 5. | E | 15. | B | 25. | B |
| 6. | D | 16. | E | 26. | B |
| 7. | A | 17. | C | 27. | C |
| 8. | D | 18. | C | 28. | E |
| 9. | B | 19. | B | | |
| 10. | B | 20. | A | | |

EXAM II SECTION I PART A

| | | | | | |
|---|---|---|---|---|---|
| 1. | C | 11. | C | 21. | D |
| 2. | A | 12. | B | 22. | A |
| 3. | B | 13. | A | 23. | C |
| 4. | C | 14. | D | 24. | B |
| 5. | D | 15. | E | 25. | B |
| 6. | C | 16. | C | 26. | A |
| 7. | E | 17. | C | 27. | A |
| 8. | A | 18. | E | 28. | E |
| 9. | B | 19. | B | | |
| 10. | D | 20. | D | | |

EXAM I SECTION I PART B

| | | | | | |
|---|---|---|---|---|---|
| 1. | D | 7. | B | 13. | C |
| 2. | C | 8. | C | 14. | D |
| 3. | B | 9. | C | 15. | B |
| 4. | E | 10. | C | 16. | D |
| 5. | D | 11. | E | 17. | E |
| 6. | D | 12. | E | | |

EXAM II SECTION I PART B

| | | | | | |
|---|---|---|---|---|---|
| 1. | E | 7. | E | 13. | D |
| 2. | D | 8. | A | 14. | D |
| 3. | D | 9. | A | 15. | C |
| 4. | C | 10. | A | 16. | C |
| 5. | C | 11. | D | 17. | D |
| 6. | B | 12. | A | | |

EXAM I SECTION II PART A

1. a) $\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}$ b) $2\pi$ c) $v = 4\pi,\ a = 0$

2. a) $(1, 5)$ b) $(-1, 5)$

   c) $A = \int_{-2}^{1} (-x+6-f(x))\,dx = 6.75$

   $B = \int_{-1}^{1} (5-f(x))\,dx = 4 \qquad A - B = 2.75$

   $\frac{4}{2.75} = \frac{16}{11}$

3. a) $c < \frac{1}{2}$ b) $\frac{16}{9}$ units$^2$ c) $\frac{4}{9}\pi$ units$^2$

EXAM II SECTION II PART A

1. a) 1.764 units$^2$ b) 30.460 units$^2$

   c) 3.671 units$^2$

2. a) $2\pi\sqrt{2}$ in/sec b) $8(\pi - 2)$in$^2$/sec

3. a) $\int_0^2 \sqrt{1+e^{-x}}\,dx$

   b) 2.39 units$^2$

   c) $\pi\left(3 - \frac{1}{e^2}\right) \approx 8.9996$units$^3$

EXAM I SECTION II PART B

4. c) $y = x - 1$

   d) $y = x - 1 + Ce^{-x} \Rightarrow x - y = 1 - Ce^{-x}$

   $\frac{dy}{dx} = \frac{d(x - 1 + Ce^{-x})}{dx} = 1 - 0 + Ce^{-x}$

   $= 1 + Ce^{-x}$

5. a) $y = 2x - 5$ b) $x = 1$ c) 0.75

   d) $x = -3, -1, 3$ e) $x = 4$

6. a) $A = \sin(\text{Arccos } k) - k \text{ Arccos } k$

   b) $\frac{\sqrt{3}}{2} - \frac{\pi}{6}$ units$^2$ c) $\frac{dA}{dt} = -\frac{1}{3}$ units$^2$/sec

EXAM II SECTION II PART B

4. a) (0, 1/10) has an inflection point

   b) $P = \frac{3}{1 + 2e^{-6t}}$ c) $t = \frac{\ln 2}{6}$

5. a) $(-3, 1)$ and $(3, 4)$

   b) $x = -3, x = 1, x = 3, x = 4$

   c) $(-2, 2)$

6. a) $x < -\sqrt{\frac{b}{a}}$ or $x > \sqrt{\frac{b}{a}}$

   b) Rel max at $\left(-\sqrt{\frac{b}{a}}, -2\sqrt{ab}\right)$

   Rel min at $\left(\sqrt{\frac{b}{a}}, 2\sqrt{ab}\right)$

   c) $x > 0$ d) no inflection point.

# Answers

EXAM III SECTION I PART A

| | | | | | |
|---|---|---|---|---|---|
| 1. | B | 11. | A | 21. | D |
| 2. | C | 12. | A | 22. | C |
| 3. | E | 13. | C | 23. | E |
| 4. | E | 14. | D | 24. | A |
| 5. | C | 15. | B | 25. | D |
| 6. | C | 16. | E | 26. | B |
| 7. | D | 17. | C | 27. | C |
| 8. | B | 18. | C | 28. | C |
| 9. | C | 19. | A | | |
| 10. | E | 20. | D | | |

EXAM IV SECTION I PART A

| | | | | | |
|---|---|---|---|---|---|
| 1. | A | 11. | E | 21. | B |
| 2. | E | 12. | B | 22. | C |
| 3. | D | 13. | D | 23. | D |
| 4. | B | 14. | D | 24. | A |
| 5. | C | 15. | C | 25. | A |
| 6. | C | 16. | E | 26. | C |
| 7. | E | 17. | D | 27. | D |
| 8. | A | 18. | D | 28. | D |
| 9. | D | 19. | A | | |
| 10. | B | 20. | D | | |

EXAM III SECTION I PART B

| | | | | | |
|---|---|---|---|---|---|
| 1. | A | 7. | C | 13. | E |
| 2. | B | 8. | D | 14. | C |
| 3. | B | 9. | C | 15. | A |
| 4. | D | 10. | C | 16. | B |
| 5. | E | 11. | C | 17. | E |
| 6. | C | 12. | B | | |

EXAM IV SECTION I PART B

| | | | | | |
|---|---|---|---|---|---|
| 1. | D | 7. | B | 13. | B |
| 2. | D | 8. | B | 14. | C |
| 3. | E | 9. | D | 15. | B |
| 4. | D | 10. | D | 16. | D |
| 5. | B | 11. | B | 17. | B |
| 6. | B | 12. | B | | |

EXAM III SECTION II PART A

1. a) 1.146 units$^2$ (denote this as $a$)

   b) $\int_0^a \pi(9\cos^2 x - e^{2x^2})\, dx$

   c) $\int_0^a (3\cos x - e^{x^2})^2\, dx$

2. a) (0, 0), (0.964, 0), (1.684, 0)

   b) (0, 0.398), (1.351, 3)

   c) min = $-0.098$, max = 1.366

3. a) 8 units$^2$ b) $\frac{2\sqrt{3}}{3} \approx 1.155$

EXAM IV SECTION II PART A

1. a) $y = 110e^{-0.1t} + 70$

   b) 110.467° c) $t \geq 7.885$ min

2. a) decreasing at 1 ft/sec

   b) distance is incr. at 1.471 ft/sec

3. a) 8 b) 4480

   c) $(-1, 1)$ and $(4, \infty)$

   d) (2, 2.953) and (−0.526, 1.288)

EXAM III SECTION II PART B

4. a) $p = -2, q = 5$ b) $p = -6$

   c) $p^2 < 3q$

5. a) $s(6) = 3$ km b) only at $t = 4$ min

6. a) $\frac{y}{y^2 - x}$

   b) At (1,2), $\frac{dy}{dx} = \frac{2}{3}$

   Tangent line: $y - 2 = \frac{2}{3}(x - 1)$

   c) $\frac{d^2y}{dx^2} = -\frac{4}{27}$

EXAM IV SECTION II PART B

4. a) 2 b) 1 c) does not exist

   d) $\frac{3}{2}$ e) 18

5. a) even b) $\frac{4}{25}$ c) $1 - \frac{\pi}{4}$

   d) 1 e) $-1 < x < 1$

6. a) 0 b) 2

   c) $(-4, -3)$ and $(-1, 2)$ d) $x = 1$

EXAM V SECTION I PART A

| | | | | | |
|---|---|---|---|---|---|
| 1. | B | 11. | B | 21. | B |
| 2. | C | 12. | D | 22. | C |
| 3. | C | 13. | D | 23. | D |
| 4. | C | 14. | A | 24. | D |
| 5. | D | 15. | C | 25. | B |
| 6. | A | 16. | B | 26. | D |
| 7. | E | 17. | D | 27. | A |
| 8. | D | 18. | E | 28. | A |
| 9. | A | 19. | B | | |
| 10. | A | 20. | B | | |

EXAM VI SECTION I PART A

| | | | | | |
|---|---|---|---|---|---|
| 1. | A | 11. | E | 21. | C |
| 2. | C | 12. | E | 22. | C |
| 3. | B | 13. | D | 23. | C |
| 4. | E | 14. | D | 24. | B |
| 5. | C | 15. | D | 25. | C |
| 6. | A | 16. | A | 26. | D |
| 7. | C | 17. | A | 27. | C |
| 8. | B | 18. | E | 28. | E |
| 9. | D | 19. | B | | |
| 10. | B | 20. | C | | |

EXAM V SECTION I PART B

| | | | | | |
|---|---|---|---|---|---|
| 1. | E | 7. | D | 13. | C |
| 2. | C | 8. | B | 14. | E |
| 3. | B | 9. | E | 15. | D |
| 4. | E | 10. | B | 16. | D |
| 5. | D | 11. | E | 17. | B |
| 6. | C | 12. | D | | |

EXAM VI SECTION I PART B

| | | | | | |
|---|---|---|---|---|---|
| 1. | E | 7. | A | 13. | A |
| 2. | C | 8. | B | 14. | D |
| 3. | C | 9. | B | 15. | B |
| 4. | C | 10. | D | 16. | C |
| 5. | E | 11. | C | 17. | B |
| 6. | D | 12. | E | | |

EXAM V SECTION II PART A

1. a) 8.3176

b) $v(1) = e - \frac{1}{2} > 0$ so it moves to the right at $e - \frac{1}{2} \approx 2.218$ ft/sec.

c) $t > 0.1756$ d) $x(.1756) = 0.7729$

2. a) $L = 39$ m$^3$/min, $U = 49$ m$^3$/min

b) max error = 5 m$^3$/min c) 14.667 m$^3$

3. a) $2 - \frac{2}{e^k}$ units$^2$ b) $2\pi\left[1 - \frac{1}{e^{2k}}\right]$ units$^3$

c) $2\pi$ units$^3$

EXAM VI SECTION II PART A

1. a) 36 b) 3.375 c) 192

2. a) 1.5 b) $s(t) = 1 + t + \ln\sqrt{1+t^2}$ =161

c) 1 d) 95.441

3. a) 47.4 metric tons of air pumped out

b) Yes, because near $t = 4$,
$R'$ is positive on $0 < t < 4$ and
$R'$ is negative on $4 < t < 8$.

c) 5.833 metric tons per hour

EXAM V SECTION II PART B

4. a) $y = C\sqrt{x^2+4}$

b) $y = 2\sqrt{x^2+4}$ c) $x \in \Re$, $y \geq 4$

5. a) $x = 1, x = 3, x = 4.5$

b) (3, 3) c) $1 < x < 3$ or $4.5 < x < 5$

6. a) $x < -1$ or $x > 0$ b) $\frac{1}{x(x+1)}$

c) $y - \ln 0.5 = 0.5(x-1)$

d) $\frac{e^x}{(1-e^x)^2}$

EXAM VI SECTION II PART B

4. a) 24 ft/sec$^2$ b) $0 < t < 7$
c) 336 ft d) 448 ft

5. b) $b = -2$

c) $g'(x) = 2x - y$ and $g'(0) = 0$
$g''(x) = 2 - y' = 2 - 2x + y$
at $(0, 0)$ $g''(0) = 2$ $\therefore$ rel. min.

6. a) The graph of $H$ is the graph of $G$ moved up 3.5 units.
b) $(-5, -3)$ and $(1, 5)$
c) $x = -3$
d) $(-2, 1)$ and $(1, 3)$

# Index



The *multiple-choice* questions and the *free-response* questions are listed in separate sections, each of which is divided into three broad categories:

I. Differentiation,
II. Integration,
III. Continuity, Limits & Graphs.

The problem type called *Graph Stem* refers to problems in which functions are defined graphically.

## MULTIPLE-CHOICE

(Calculator-active problems are labeled **C**.)


**I. DIFFERENTIATION**

**Applications** p10#27; p109#7C
**Average Rate Of Change** p34#26; p59#2C; p63#11C; p85#8C
**Definition Of The Derivative & Theorems** p11#1C; p16#15C; p26#3; p31#18; p35#1C; p37#7C; p50#5; p51#7; p75#6; p98#4; p114#16C; p124#10; p134#8C p136#12C
**Graph Stems** p2#3; p9#26; p11#1C; p12#5C; p37#6C, #7C; p39#13C; p41#17C; p62#8C; p63#12C; p80#21; p83#1C; p84#3C; p86#9C; p87#12C; p104#21; p105#24; p113#15C; p121#2; p124#11; p125#13; p128#23; p131#1C; p132#4C
**Implicit Differentiation** p12#4C; p30#15; p34#28; p56#22; p65#17C; p74#5; p75#8; p88#14C; p105#25; p112#12C; p123#6
**Increasing & Decreasing** p1#1; p4#11; p10#28; p12#5C; p14#11C; p33#25; p41#17C; p56#23; p59#1C; p62#8C; p80#21,#23; p81#25; p83#1C; p84#3C; p86#9C; p102#17; p108#3C; p110#9C; p113#15; p128#22
**Maximum & Minimum** p2#5; p4#11; p13#6C; p32#22; p39#13C; p40#15C; p55#20; p57#25; p59#1C; p78#15; p79#19; p86#9C; p86#11C; p99#6; p100#11; p101#13; p102#17; p108#3C; p110#8C; p113#14C; p126#15; p132#5C; p137#14C
**Product, Quotient & Chain Rules** p1#2; p2#3, #5; p6#16; p7#19, #20; p8#22; p11#2C; p13#8C; p15#12; p16#14C; p28#11; p29#13; p30#16; p31#19; p32#21; p36#4C, #5C; p38#11C; p52#11; p54#15; p58#28; p65#15C; p74#3; p78#17; p97#1; p103#18,#20; p109#6C; p126#16; p129#24; p130#28; p134#8C; p138#16C
**Related Rates** p10#27; p37#6; p55#18; p63#12C; p76#10; p87#12C; p102#15; p109#7C; p130#28; p132#3C
**2nd Derivative, Concavity & Inflection Points** p3#7; p12#4C; p13#8C; p26#4; p39#13C; p40#14C; p41#17C; p51#8; p58#28; p59#1C; p62#8C; p77#16; p80#21,#23; p83#1C; p84#3C; p86#9C; p99#6; p101#13; p102#17; p104#24; p107#1C; p110#8C,#9C; p113#14C; p121#1; p124#11; p127#19; p128#23; p132#4C
**Slope Fields**..p5#14; p33#23; p53#14; p79#20..p97#2;..126#17
**Tangents & Normals** p1#2; p3#7; p7#20; p9#25; p12#5C; p25#1; p32#21; p36#3C; p39#13; p40#15C; p49#1; p53#13; p63#11C; p64#14C; p74#5; p75#7; p78#17; p84#5C; p87#13C; p98#3; p105#24,#25; p127#18; p131#2C
**Velocity & Acceleration** p9#24, #26; p14#11C; p17#16C; p27#6; p34#28; p58#27; p64#13C; p82#26; p85#6C; p104#23; p125#14; p132#5C

**II. INTEGRATION**

**Applications** p14#9C; p17#16; p35#2C; p63#12C; p81#24; p107#2C; p108#3C; p124#9; p129#26
**Area** p6#17; p13#7C; p26#5; p30#17; p37#6C; p39#12C; p50#4; p60#4C; p62#9C; p81#25; p85#7C; p86#10C; p105#26; p138#17C
**Average Value** p5#13; p33#24; p52#9; p76#11; p100#10; p136#13C; p137#15C
**Definite Integrals** p2#4; p8#21; p12#3C; p25#2; p27#8; p28#10; p29#14; p35#2C; p37#8C; p49#2; p51#6; p55#19; p56#21; p59#2C; p77#13; p89#16; p101#14; p102#16; p122#3, #5; p123#7; p127#20
**Differential Equations** p8#23; p14#9C; p29#12; p35#2C; p38#10C; p65#16C; p81#24; p89#17C; p106#27; p107#2C; p111#10C; p128#21; p133#6C; p134#9C

## Index

**Fundamental Theorem** p5#12; p17#17C; p34#27; p37#8C; p53#12; p54#17; p59#2C; p62#10C; p74#4; p82#28; p85#8C; p106#28; p111#11C; p113#15C; p121#2; p125#12; p133#7C
**Graph Stems** p14#10; p37#6C, #7C; p57#26; p61#6C; p63#11C; p89#16C; p104#21; p113#15; p124#9;
**Numerical Integration** p3#6; p14#10C; p37#7C; p57#26; p60#3C; p63#11C; p80#22; p108#4C; p112#13C; p135#10C
**Symbolic Integration** p3#8; p7#18; p15#13C; p27#7; p33#23; p73#2; p82#27; p84#4C; p101#12; p103#19
**Velocity & Acceleration** p14#11C; p17#16C; p41#16C; p52#10; p61#7C; p76#9; p99#7; p114#17C
**Volume** p4#10; p9#24; p14#10C; p38#9C; p57#24; p61#7C; p79#18; p88#15C; p108#5C; p135#11C

**III. CONTINUITY, LIMITS, GRAPHS**
**Continuity** p4#11; p16#15C; p31#20; p36#4C; p50#3; p62#8C; p77#14; p83#2C; p84#5C; p98#4; p100#9; p114#16C; p122#4; p129#25; p131#1C
**Limits** p4#9; p6#15; p11#1C; p26#3; p28#9; p36#4C; p50#5; p54#16; p60#5C; p73#1; p75#6; p77#12, #14;; p99#8; p100#9; p123#8

**FREE-RESPONSE**
(Problems #1, #2, and #3 are calculator-active. No calculators are used on #4, #5, and #6.)

**I. DIFFERENTIATION**
**Average Rate Of Change** p19#1; p116#1
**Graph Stems** p23#5; p47#5; p71#5; p94#4; p120#5; p145#6
**Implicit Differentiation** p72#6
**Increasing & Decreasing** p47#5; p48#6; p68#2; p70#4; p93#3; p143#4
**Maximum & Minimum** p47#5; p48#6; p68#2; p69#3; p96#6; p120#5; p145#6
**Product, Quotient & Chain Rules** p95#5; p120#5; p121#6
**Related Rates** p24#6; p44#2; p92#2
**2nd Derivative, Concavity & Inflection Points** p20#2; p23#5; p47#5; p48#6; p70#4; p72#6; p96#6; p145#6
**Tangents & Normals** p19#1; p20#2; p69#3; p72#6: p121#6
**Velocity & Acceleration** p19#1; p71#5; p116#1; p143#4

**II. INTEGRATION**
**Applications** p91#1
**Area** p20#2; p21#3; p24#6; p43#1; p45#3; p67#1; p69#3; p118#3; p140#1C
**Average Value** p19#1; p142#3C
**Definite Integrals** p94#4; p95#5; p118#3
**Differential Equations** p91#1; p119#4
**Graph Stems** p96#6; p117#2
**Numerical Integration** p45#3; p94#4; p117#2; p
**Slope Fields** p22#4; p46#4; p144#5
**Velocity & Acceleration** p71#5; p141#2; p143#4
**Volume** p21#3; p43#1; p45#3; p67#1; p118#3; p140#1C

**III. CONTINUITY, LIMITS, GRAPHS**
**Graphs** p71#5; p120#5
**Limits** p94#4; p141#2C